DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE
(AUTONOMOUS)
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Department of Aeronautical Engineering

U20AE602 VIBRATION AND ELEMENTS OF AEROELASTICITY

QUESTION BANK
PART A

Q. 1.What are the two degree of freedom system?

Ans. The system which requires two co-ordinates to describe its motion completely is called two degree of freedom system. In a two degree of freedom system there are two masses which have two natural frequencies and two co-ordinates are required to specify the configuration of the system completely.

Q. 2. Define 'Normal mode of vibration'?

Ans. In a two degree freedom system there are two natural frequencies of the system. The system at its lowest or first natural frequency its first and next higher i.e. second natural frequency is called its second mode. If the two masses vibrate at same frequency and in phase it is called principal mode of vibration. If prncipa1 mode of vibration the amplitude of one of the masses is then it is known as normal mode of vibration.

Q.3. Draw the mode shapes for two rotor system.

Ans.

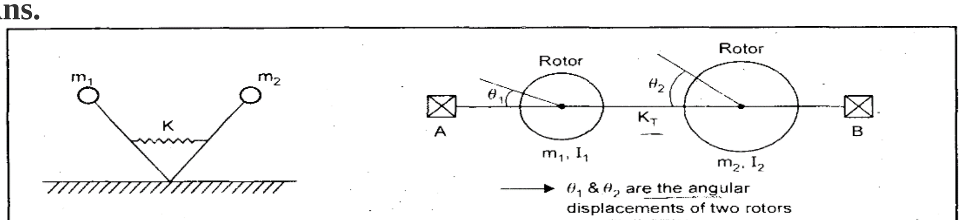


Fig. 5.1 Fig. 5.2

Torsional Vibrations : Consider fig. 5.2. A shaft AB is carrying two rotors of moment of inertia I and ‘2 Let and °2 be the angular displacements of rotor at any .instant from mean position. The equation of motion can be written s, any .instant from mean position. The equation of motion can be written s,

$$I_1 \ddot{\theta}_1 = -K_T(\theta_1 - \theta_2) \quad \text{...(I)}$$

$$I_1 \ddot{\theta}_1 + K_T(\theta_1 - \theta_2) = 0 \quad \text{...(II)} \quad \left[\ddot{\theta}_1 = \frac{d^2\theta_1}{dt^2} \right]$$

$$I_2 \ddot{\theta}_2 = -K_T(\theta_2 - \theta_1) \quad \text{...(III)}$$

$$I_2 \ddot{\theta}_2 + K_T(\theta_2 - \theta_1) = 0 \quad \text{...(IV)} \quad \left[\ddot{\theta}_2 = \frac{d^2\theta_2}{dt^2} \right]$$

Put,

$$\theta_1 = a_1 \sin \omega t, \theta_2 = a_2 \sin \omega t$$

$$\ddot{\theta}_1 = -a_1 \omega^2 \sin \omega t$$

$$\ddot{\theta}_1 = -\omega^2 \theta_1, \text{ similarly, } \ddot{\theta}_2 = -\omega^2 \theta_2$$

Putting these values in (II) and (IV)

$$I_1 \times -\omega^2 \theta_1 + K_T(a_1 - a_2) \sin \omega t = 0$$

$$-I_1 \omega^2 a_1 \sin \omega t + K_T(a_1 - a_2) \sin \omega t = 0$$

$$-\omega^2 I_1 a_1 + K_T(a_1 - a_2) = 0 \quad \text{...(V)}$$

$$I_2 \times -\omega^2 \theta_2 + K_T(a_2 - a_1) \sin \omega t = 0$$

$$I_2 \times -\omega^2 \times a_2 \sin \omega t + K_T(a_2 - a_1) \sin \omega t = 0$$

$$I_2 \omega^2 a_2 + K_T(a_2 - a_1) = 0 \quad \text{...(VI)}$$

$$\omega^2 I_1 a_1 + K_T(a_1 - a_2) = 0$$

$$\omega^2 I_2 a_2 + K_T(a_2 - a_1) = 0$$

$$(K_T - I_1\omega^2) a_1 - K_T a_2 = 0$$

$$-K_T a_1 + (K_T - I_2\omega^2) a_2 = 0$$

Solving by determinant

$$\begin{vmatrix} K_T - I_1\omega^2 & -K_T \\ -K_T & K_T - I_2\omega^2 \end{vmatrix} = 0$$

$$(K_T - I_1\,\omega^2)\,(K_T - I_2\,\omega^2) - K_T^2 = 0$$
$$K_T^2 - K_T\,I_2\,\omega^2 - K_T\,I_1\,\omega^2 + I_1\,I_2\,\omega^4 - K_T^2 = 0$$
$$\omega^2\,(I_1\,I_2\,\omega^2 - K_T\,I_1 - K_T\,I_2) = 0$$
$$\omega^2 = 0$$

$$\boxed{\omega_1 = 0} \qquad \text{...(1)}$$

$$I_1\,I_2\,\omega^2 - K_T\,(I_1 + I_2) = 0$$

$$\omega_2 = \sqrt{\frac{K_T\,(I_1 + I_2)}{I_1 I_2}}\ \text{rad/s} \qquad \text{...(2)}$$

Put value of $\omega_1 = 0$ in (VII)

$$a_1\,K_T - K_T\,a_2 = 0$$

$$\boxed{\frac{a_1}{a_2} = 1} \qquad \text{...(IX)}$$

From (VIII)

$$\frac{a_1}{a_2} = \frac{K_T - I_2\omega^2}{K_T}$$

$$\frac{a_1}{a_2} = I - \frac{I_2\omega^2}{K_T}$$

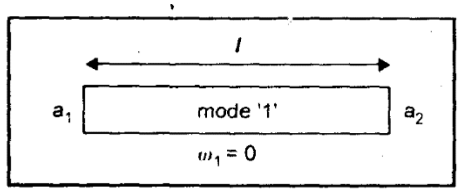
Put value of $\omega_2 = \sqrt{\dfrac{K_T\,(I_1 + I_2)}{I_1 I_2}}$ rad/s

$$\frac{a_1}{a_2} = 1 - \frac{I_2}{K_T} \times \frac{K_T(I_1 + I_2)}{I_1\,I_2}$$

$$\frac{a_1}{a_2} = 1 - \left(1 + \frac{I_2}{I_1}\right) = \frac{-I_2}{I_1}$$

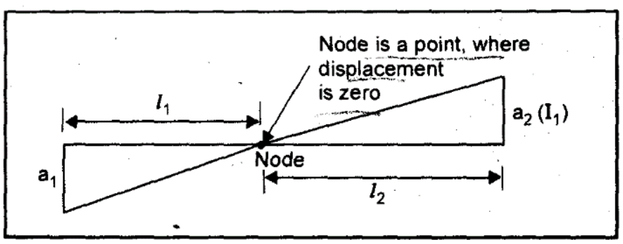
It shows that the angular displacements of rotors are inversely proportional to their inertia.
The section of the shaft where angular displacement is zero Is known as node. First Mode shape

$$\omega_1 = 0, \frac{a_1}{a_2} = 1$$



Second Mode Shape

$$\omega_2 = \sqrt{K_T \frac{(I_1 + I_2)}{I_1 I_2}}, \frac{a_1}{a_2} = \frac{-I_2}{I_1}$$



$$\omega = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s}$$

Q. 4. What is a semi definite system? Determine the frequencies of the system?

Ans. Semi-Definite Systems : The system having one of their natural frequencies equal to zero are known as semi-definite systems. The example of the type of system is when two masses m1 and m2 are connected by spring K

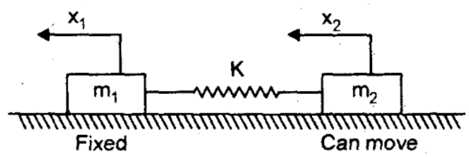


Fig. 5.3

$$m_1 \ddot{x}_1 = -K(x_1 - x_2)$$

$$m_1 \ddot{x}_1 + K(x_1 - x_2) = 0 \qquad \text{...(I)}$$

$$m_2 \ddot{x}_2 = -K(x_2 - x_1)$$

$$m_2 \ddot{x}_2 + K(x_2 - x_1) = 0 \qquad \text{...(II)}$$

$$x_1 = A_1 \sin[(\omega t) + \phi]$$
$$x_2 = A_2 \sin[(\omega t) + \phi]$$
$$\ddot{x}_1 = -A_1 \omega^2 \sin[(\omega t) + \phi]$$
$$\ddot{x}_2 = -A_2 \omega^2 \sin[(\omega t) + \phi]$$

$$m_1 \times -A_1 \omega^2 \sin(\omega t + \phi) + K(A_1 - A_2)\sin(\omega t + \phi) = 0$$
$$(K - m_1 \omega^2) A_1 - KA_2 = 0 \qquad \text{...(III)}$$
$$m_2 \times -A_2 \omega^2 \sin(\omega t + \phi) + K(A_2 - A_1)\sin(\omega t + \phi) = 0$$
$$-KA_1 + (K - m_2 \omega^2) A_2 = 0 \qquad \text{...(IV)}$$

The equations (III) and (IV) will have non-trivial solutions if ;

$$\begin{vmatrix} K - m_1\omega^2 & -K \\ -K & K - m_2\omega^2 \end{vmatrix} = 0 \qquad \text{[Take determinant of following matrix]}$$

$$(K - m_1 \omega^2)(K - m_2 \omega^2) - K^2 = 0$$
$$K^2 - m_2 K\omega^2 - m_1 K\omega^2 + m_1 m_2 \omega^4 - K^2 = 0$$
$$m_1 m_2 \omega^4 - m_1 K \omega^2 - m_2 K \omega^2 = 0$$

$$\omega^4 - \frac{K}{m_2}\omega^2 - \frac{K}{m_1}w^2 = 0 \qquad \left[\left(\omega^2 - \frac{K}{m_2} - \frac{K}{m_1}\right) = 0 \right]$$

$$\omega^2\left(\omega^2 - \frac{K}{m_2} - \frac{K}{m_1}\right) = 0$$

$$\omega_1 = 0; \qquad \omega = \sqrt{K\left(\frac{1}{m_2} + \frac{1}{m_1}\right)}$$

$$\omega_2 = \sqrt{\frac{K\,(m_1 + m_2)}{m_1 m_2}} \text{ rad/s.}$$

Q. 5. What is co-ordinate coupling? Determine the natural frequencies of such system with dynamic coupling? (V.V. Imp.)

Ans. Co-ordinate coupling. When we apply brakes on a automobile two motions of car body occur simultaneously.
(1) Translatory (x)
(2)angular.
This type of unbalance occurs on the system because centre of gravity (C) of car and centre of rotation do not coincide.
m—Massofcar
I-*MOI
x — Translatory motion
0 — Angular Motion
Equation of motion can be written as :

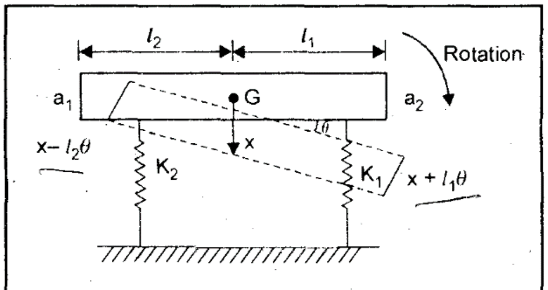


Fig. 5.4

$$m\ddot{x} = -K_2(x - l_2\theta) - K_1(x + l_1\theta)$$

$$m\ddot{x} + (K_1 + K_2)x - (K_2 l_2 - K_1 l_1)\theta = 0$$

$$I\ddot{\theta} = K_2(x - l_2\theta)l_2 - K_1(x + l_1\theta)l_1$$

$$I\ddot{\theta} = (K_2 l_2 - K_1 l_1)x + (K_2 l_2^2 + K_1 l_1^2)\theta = 0 \qquad \text{...(II)}$$

= n (I) and (II) are coupled equations as both equations contain x and 0 terms
If

$$K_1 l_1 = K_2 l_2 \text{ then ;}$$

$$m\ddot{x} + (K_1 + K_2)x = 0 \qquad \text{...(III)} \quad \text{[complete translatory equation]}$$

$$I\ddot{\theta} + (K_2 l_2^2 + K_1 l_1^2)\theta = 0 \qquad \text{...(IV)} \quad \text{[oscillatory equation]}$$

Equation III is of translator nature.
Equation IV is of oscillator nature.

These are uncoupled differential equations and when $K_1 l_1 = K_2 l_2$ then it is called dynamic coupling.
The natural frequencies of the system are:

$$\omega_1 = \sqrt{\frac{K_1 + K_2}{m}} \text{ rad/s}, \quad \omega_2 = \sqrt{\frac{K_1 l_1^2 + K_2 l_2^2}{I}} \text{ rad/s}$$

Q. 6. What are vibration absorbers ? Prove that spring force of the absorber system is equal and opposite to the excitition force for main system to be stationary?

Ans. Vibration Absorber. When a structure which is excited by an external harmonic force has undesirable vibrations, it becomes necessary to eliminate them by coupling some vibrating system to it. The vibrating system is known as vibration absorber or dynamic vibration absorber. Vibration absorbers are used to control the structural resonance (consider the main figure)

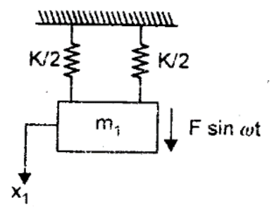


Fig. 5.5 Main System

The natural frequency of this system is

$$\sqrt{\frac{K}{m_1}}.$$

When forcing frequency (0 becomes equal to natural frequency of main system then resonance takes place. In order to reduce the amplitude of mass ‘nz1’ it is coupled with spring mass system (m2 — K2) called Vibration absorber. The spring mass system (ni2 — K2) will acts as vibration absorber andies the amplitude of ni1 to zero if its natural frequency is equal to the excitation frequency

$$\omega = \sqrt{\frac{K_2}{m_2}}.$$

Then, when $\frac{K_1}{m_1} = \frac{K_2}{m_2}$ the absorber is called **tuned absorber.**

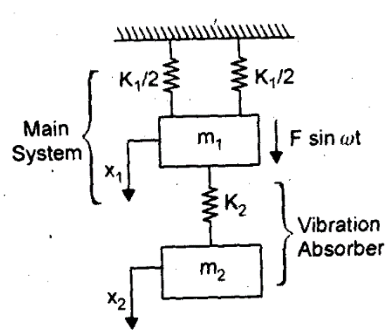


Fig 5.6

Equations of Motion

$$m_1 \ddot{x}_1 = -K_1 x_1 - K_2 (x_1 - x_2) + F \sin \omega t$$

$$m_1 \ddot{x}_1 + (K_1 + K_2)x_1 - K_2 x_2 = F \sin \omega t \qquad \text{...(I)}$$

$$m_2 \ddot{x}_2 = -K_2 (x_2 - x_1)$$

$$m_2 \ddot{x}_2 + K_2 (x_2 - x_1) = 0 \qquad \text{...(II)}$$

$$x_1 = A_1 \sin \omega t$$
$$x_2 = A_2 \sin \omega t$$

$$m_1 \times -\omega^2 A_1 \sin \omega t + (K_1 + K_2) A_1 \sin \omega t - K_2 A_2 \sin \omega t = F \sin \omega t$$

$$A_1 (K_1 + K_2 - m_1 \omega^2) - K_2 A_2 = F \qquad \text{...(III)}$$

$$m_2 \times -A_2 \omega^2 \sin \omega t - + K_2 (A_2 - A_1) \sin \omega t = 0$$

$$-K_2 A_1 + (K_2 - m_2 \omega^2) A_2 = 0 \qquad \text{...(IV)}$$

Solving (III) and (IV) for A_1 and A_2. From (IV)

$$A_2 = \frac{K_2 A_1}{K_2 - m_2 \omega^2}$$

Putting in (III)

$$A_1 (K_1 + K_2 - m_1 \omega^2) - \frac{-K_2^2 A_1}{K_2 - m_2 \omega^2} = F$$

$$A_1 [K_1 + K_2 - m_1 \omega^2] [K_2 - m_2 \omega^2] - K_2^2 A_1 = F (K_2 - m_2 \omega^2)$$

$$A_1 [K_1 K_2 - K_1 m_2 \omega^2 + K_2^2 - m_2 \omega^2 K_2 - m_1 K_2 \omega^2 + m_1 m_2 \omega^4 - K_2^2] = F (K_2 - m_2 \omega^2)$$

$$\boxed{A_1 = \frac{F(K_2 - m_2 \omega^2)}{\beta}} \qquad \text{...(V)}$$

Where

$$\beta = [m_1 m_2 \omega^4 - [m_1 K_2 + m_2 (K_1 + K_2) \omega] + k_1 k_2]$$

$$\boxed{A_2 = \frac{F K_2}{\beta}} \qquad \text{...(VI)}$$

In order that amplitude of mass ni1 is zero

Put A1 =0 (so that mass rn1 must not vibrate)

$$K_2 = m_2 \omega^2$$

$$\boxed{\omega = \sqrt{\frac{K_2}{m_2}} = \omega_2}$$

The vibration absorber in which mass and spring constant are selected such that the above condition is satisfied becomes **dynamic vibration absorber.**

Let us assume

$$A_{st} = \frac{F}{K_1} = \text{Static deflection or zero frequency deflection}$$

$$\omega_1 = \sqrt{\frac{K_1}{m_1}} = \text{Natural frequency of main system}$$

$$\omega_2 = \sqrt{\frac{K_2}{m_2}} = \text{Natural frequency of vibration absorber}$$

$$\mu_1 = \frac{m_2}{m_1} = \text{Mass ratio}$$

Multiply N^r and D^r *by* $K_1 K_2$

$$A_1 = \frac{F(K_2 - m_2\omega^2)}{K_1 K_2} \Bigg/ \frac{[m_1 m_2 \omega^4 - [m_1 K_2 + m_2(K_1 + K_2)]\omega^2] + K_1 K_2]}{K_1 K_2}$$

$$A_1 = \frac{F/K_1 \left(1 - \frac{\omega^2}{\omega_2^2}\right)}{\frac{\omega^4}{\omega_1^2 \omega_2^2} - \left[\frac{m_1}{K_1} + m_2\left(\frac{1}{K_2} + \frac{1}{K_1}\right)\right]\omega^2 + 1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2}-\left(\frac{m_1}{K_1}+\frac{m_2}{K_2}+\frac{m_2}{K_1}\right)\omega^2+1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\,\omega_2^2}-(\omega_1^{-2}+\omega_2^{-2}+\mu\omega_1^{-2})\omega^2+1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\,\omega_2^2}-\left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VII)}$$

Simlilarly

$$\frac{A_2}{A_{st}} = \frac{K_1K_2}{\beta} \qquad \left[A_2 = \frac{F}{K_1}\ \frac{K_1K_2}{\beta} \text{ where } \frac{F}{K_1}=X_{st}\right]$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^4}{\omega_1^2\,\omega_2^2}-\left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VIII)}$$

when A1 0, from VII

$$\boxed{\omega = \omega_2}$$

At $A_1 = 0, A_2 = ?$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2} + 1\right] + 1}$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - (1+\mu)\frac{\omega^2}{\omega_1^2}} = \frac{1}{-\mu\frac{\omega^2}{\omega_1^2}}$$

$$\frac{A_2}{A_{st}} = \frac{-\omega_1^2}{\mu\omega^2}$$

$$A_2 = A_{st} \times -\frac{\omega_1^2}{\mu\omega^2} = A_{st} \times \frac{-\omega_1^2}{\mu\omega_2^2}$$

$$A_2 = \frac{-F}{K_1}\,\frac{K_1 m_2}{m_1 \mu K_2} = \frac{-F}{K_2}\,\frac{\mu}{\mu} \qquad [\because \omega = \omega_2]$$

$$\left[\mu = \frac{m_2}{m_1}\right]$$

$$\boxed{F = -A_2 K_2}$$

Hence when the amplitude A1 = 0 i.e. main system becomes stationary the spring force of the absorber is equal and opposite to exciting force. The energy of the main system is absorbed by vibration absorber which is also called auxiliary system.

Amplitude of the auxiliary system is inversely proportional to spring constant ‘K2’.

This equation is used for design of absorber.

Q.7. Discuss the effect of mass ratio on natural frequency of the vibration absorber.

Ans. We know that

$$\frac{A_1}{A_{st}} = \frac{1 - \frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2} + \frac{\omega^2}{\omega_2^2}\right] + 1} \qquad ...(i)$$

By puffing w = W2 and equating denomination of the above equation equal to 0, we Get

$$\frac{\omega^4}{\omega_2^4} - (2+\mu)\frac{\omega^2}{\omega_2^2} + 1 = 0$$

Solving, we get

$$\left(\frac{\omega}{\omega_2}\right)^2 = 1 + \frac{\mu}{2} \pm \sqrt{\mu + \frac{\mu^2}{4}} \qquad ...(ii)$$

Equation (*ii*), gives two resonant frequencies

For $\mu = 0.25$ *i.e.* $\frac{m_2}{m_1} = 0.25$ resonance occurs at frequency 0.78 and 1.28 times that of main system

For resonance

$$\omega = 0.78\,\omega_1 \text{ or } \omega = 1.28\,\omega_1$$

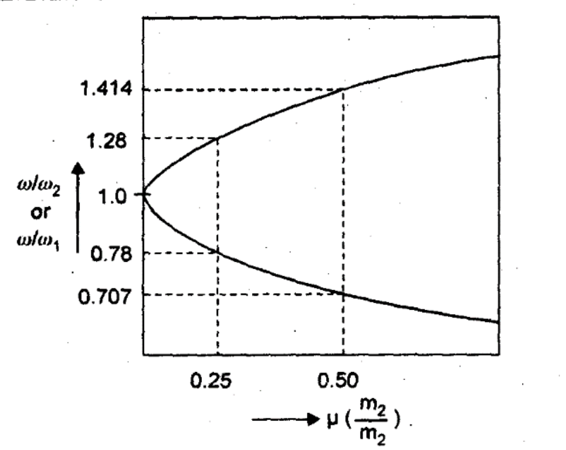


Fig. 5.7 Effect of mass ratio on natural frequency

When $\mu = 0.50$, then resonant frequencies are ; $\omega = 0.707\omega_1$ and $\omega = 1.414\,\omega_1$ *i.e.* resonant frequencies are 0.707 and 1.280 times the natural frequency of main system.

For smaller values of mass ratio (μ) the two resonant frequencies are found closer to unity *i.e.* $\omega = \omega_2 = \omega_1$

For smaller values of mass 'm_2' *i.e.* $\mu = 0$ the forcing frequency $\omega = \omega_1 = \omega_2$ *i.e.* $\omega = \omega_1$ *i.e.* natural frequency of main system becomes equal to forcing frequency and resonance takes place.

Q. 8. (i) What are the disadvantages of dynamic vibration absorber?

Ans. Demerits of Dynamic vibration Absorber : Th dynamic vibration absorber whether for the rectilinear torsional system is fully effective at a particular impressed frequency for which it is designed This means that main system will be stationary only for particular frequency. Thus dynamic vibration absorbers are effective for constant speed machine but lose their effectiveness with any change in speed of machines. Most rotors are likely to run throw' wide range speeds so dynamic vibrations absorbers become ineffective.

Q. 8. (ii) Prove that all frequency of centrifugal pendulum absorber is always proportional to the speed of rotating body.

Ans. Centrifugal Pendulum Absorber: It is effective at all speeds of rotation and is improvement over conventional dynamic vibration absorbers. A pendulum PB of length 'L' is attached to rotating member at point P which is at radius 'R' from centre of rotation 'O'. The mass of bob of pendulum is 'm' and string is assumed to have a negligible mass. The pendulum is subjected to centrifugal force which is neglected. The body is rotating with angular velocity 'w' rad/s and of centrifugal force mw2r is experienced by bob of the pendulum at radius 'r' from 0.

$$I\ddot{\theta} = -(m\omega^2 \sin \alpha)L$$

$$mL^2\ddot{\theta} + (m\omega^2 r \sin \alpha)L = 0$$

$$\boxed{\ddot{\theta} + \frac{r}{L}\omega^2 \sin\alpha = 0}$$

[$I = mL^2$ when mass of string is neglected, if mass of string/spring is considered then ; $I = \left(m + \frac{m_s}{3}\right)L^2$, where $m_s \rightarrow$ Mass of string]

$$\frac{R}{\sin\alpha} = \frac{r}{\sin(180-\theta)} \text{ (Lami's Theorem)}$$

$$\frac{R}{\sin\alpha} = \frac{r}{\sin\theta} \Rightarrow r\sin\alpha = R\sin\theta$$

Equation (I) becomes,

$$\ddot{\theta} + \frac{R}{L}\omega^2 \sin\theta = 0$$

$\sin\theta \simeq \theta$. (For small values of θ)

$$\ddot{\theta} + \frac{R}{L}\omega^2 \theta = 0$$

$$\omega_n = \sqrt{\frac{R\omega^2}{L}} \;;\; \omega_n = \omega\sqrt{\frac{R}{L}} \text{ rad/s}$$

R → Radius of rotation
L → Length of Pendulum
ω_n → Natural frequency of pendulum

$$f_n = \frac{1}{2\pi} \times \omega\sqrt{\frac{R}{L}} \text{ Hz}$$

$$f_n = N\sqrt{\frac{R}{L}} \text{ Hz}$$

N → forcing frequency in rps
$f_n \propto N$,

Natural frequency of pendulum is proportional to speed of rotational body.

Q. 8 (iii) Define order No.

Ans. Order No: The torsional system receives certain number of torques per revolution. The no. of these disturbing torques per revolution Is known as order no. of the system
2-cylinder engine working on 4-stroke - Order No. = 1
4-cylinder engine working on 4-stroke - Order No. = 2
6-cylinder engine working on 4-stroke - Order No. 3
Design of centrifugal pendulum absorber

- For pendulum absorber to be effective its natural frequency must be equal to excitation frequency or frequency of disturbing torques.

Let T sin ($n\omega t$) be the torque on I.C. engine, where $n \to$ Order No.
We know that ;

$$f_n = N\sqrt{\frac{R}{L}} \qquad ...(i)$$

where f, is Natural frequency of pendulum absorber.

For effective working of pendulum absorber $f_n = \frac{n\omega}{2\pi}$

i.e. natural frequency must he equal to excitation frequency/frequency of disturbing torque.
from (i) & (ii)

$$n\frac{\omega}{2\pi} = N\sqrt{\frac{R}{L}}$$

$$n\omega = 2\pi N\sqrt{\frac{R}{L}}$$

$$n\,\omega = \omega\sqrt{\frac{R}{L}}$$

$$n = \sqrt{\frac{R}{L}}$$

Hence Order No. $= \sqrt{\frac{R}{L}}$

$$n = \sqrt{\frac{R}{L}} \qquad ...(iii)$$

from (i)

$$f_n = N\sqrt{\frac{R}{L}}$$

$$\boxed{\frac{f_n}{N} = \sqrt{\frac{R}{N}} = n}$$

Hence order no. is ratio of natural frequency of the pendulum absorber ‘f’ in rps to the forcing frequency N’ in rps.
Order No. is also defined as:

$$n = \frac{f_n}{N} = \frac{\text{Disturbing torque impulse/sec}}{\text{Revolutions per second}}$$

$$n = \text{Disturbing torque impulse per revolution}$$

Design equation is ;

$$n = \sqrt{\frac{R}{L}}$$

for particular IC. engine is known

For known values of 'R', the length of pendulum can be calculated.

Q. 9. Write short notes on:

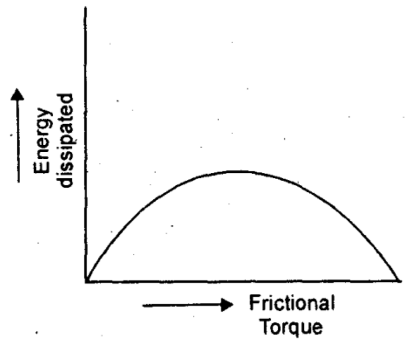
(A) Untuned Dry friction damper

(B) Untuned viscous damper

Ans. **(A) Untuned Dry friction damper or Untuned vibration Absorber (Lanchester Damper)** : This type of damper is very advantageous to use for torsional vibrations near resonance conditions. It consists of two fly wheels mounted freely over a hub. The hub is rigidly fixed to shaft undergoing vibrations. There are friction plates attached to the extension of the hub. These friction plates apply pressure on the flywheel and are responsible for driving the flywheel.

When the pressure. between the friction plates and flywheel is zero the relative velocity is maximum but frictional torque is zero. There is no energy dissipation in such case. When the pressure between the friction plate and flywheel is large due to large friction torque there is no relative velocity between flywheel and shaft and energy dissipation is zero.

When the speed of the main system is such that torsional vibrations are present in the system then the pressure between the friction material and flywheel is such that both frictional torque and relative rubbing is present then there is dissipation of energy in the absorber which causes, reduction in amplitude of the vibrations of the man system.



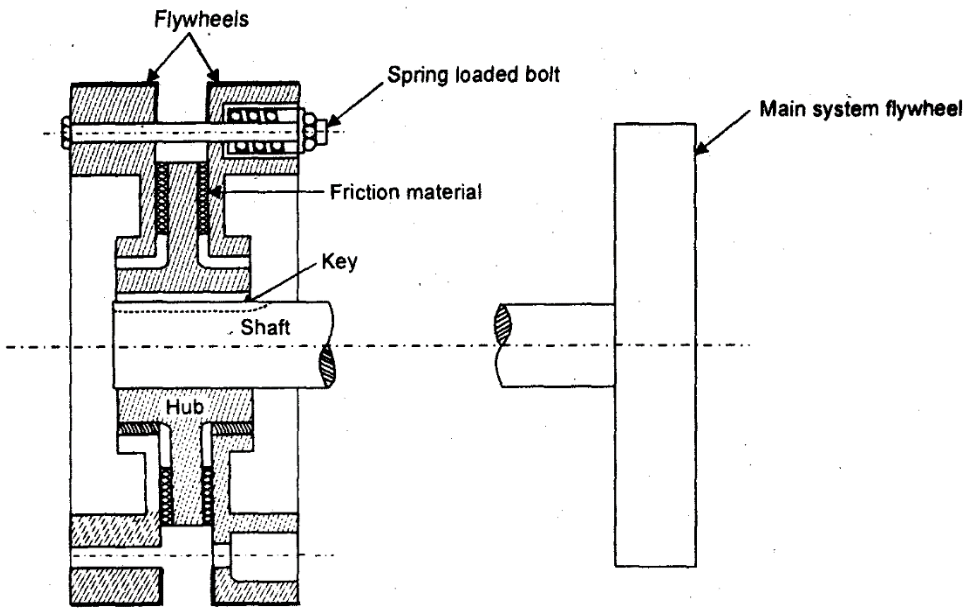


Fig. 5.9 Dry Friction Torsional Vibration absorber

(B) Untuned Viscous damper (Houdaille Damper) : this types of damper is similar in principle to the Lanchester Damper except that instead of using friction plates for dry friction damping, this system uses friction damping. It consists of a freely rotating disc enclosed in the close-fitting case which is keyed to the shaft. Normally the disc rotates at the shaft speed owing to the viscous drag of the oil between the disc and the case. However if the shaft vibrates torsionally, viscous action of the oil between the disc and casing gives a damping action.

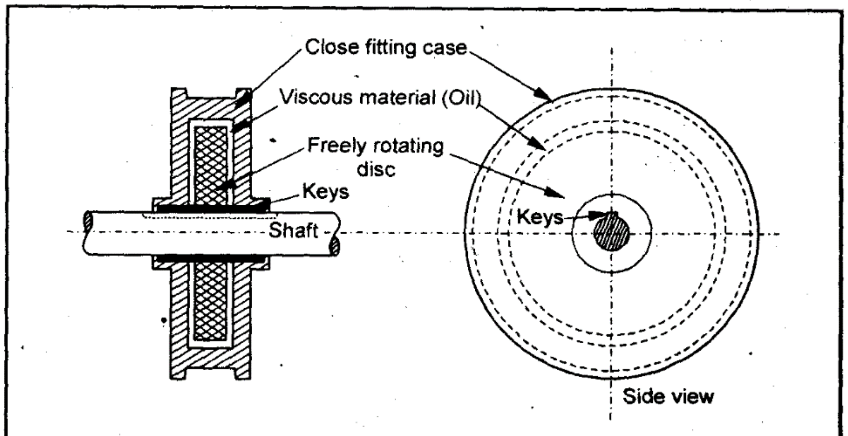


Fig 5.10. Untuned Viscous Damper (Houdaille Damper)

Q. 10. Determine the two natural frequencies of vibration and the ratio of the amplitudes of motion of mass m1 and m2 for the system shown in Fig. 5.11.

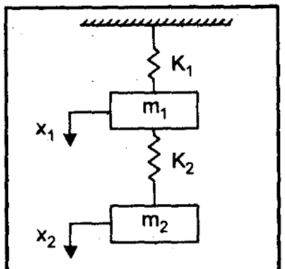


Fig. 5.11

Given : $m_1 = 1.5$ kg, $m_2 = 0.8$ kg, $K_1 = K_2 = 40$ N/m.

Ans. Let at any instant masses m_1 and m_2 are having displacements x_1 and x_2 respectively.

Equations of motion can be written as

$$m_1 \ddot{x}_1 = -k_1 x_1 - k_2 (x_1 - x_2)$$

$$m_2 \ddot{x}_2 = -k_2 (x_2 - x_1)$$

Assuming the solution of the form

$$x_1 = A_1 \sin \omega t$$

and $$x_2 = A_2 \sin \omega t$$

So above two equations can be written as

$$(k_1 + k_2 - \omega^2 m_1)A_1 - k_2 A_2 = 0$$

$$(k_2 - \omega^2 m_2) A_2 - k_2 A_1 = 0$$

$$\frac{A_1}{A_2} = \frac{k_2}{k_1 + k_2 - m_1\omega^2}$$

$$\frac{A_1}{A_2} = \frac{k_2 - m_2\omega^2}{k_2}$$

The frequency equation can be written as

$$(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2 = 0$$

or $$\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)\omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0$$

Putting $k_1 = k_2 = 40$ N/m and $m_1 = 1.5$ kg ; $m_2 = .80$ kg

$$\omega^4 - \left(\frac{80}{1.5} + \frac{40}{8}\right)\omega^2 + \frac{40 \times 40}{1.5 \times 8} = 0$$

$$\omega^4 - 103.33\,\omega^2 + 1333.33 = 0$$

$$\omega_1 = 9.39 \text{ rad/sec} \;;\; \omega_2 = 3.88 \text{ rad/sec.}$$

$$\text{The amplitude ratio} = \frac{k_2}{k_1 + k_2 - m_1\omega^2} = \frac{40}{40 + 4 - 1.5(9.39)^2} = -0.765$$

and $$\frac{A_1}{A_2} = \frac{40}{40 + 40 - 1.5(3.88)^2} = 0.696$$

Q. 11. Solve the problem shown in Fig. 5.12; m1 10 kg, m2 = 15 kg, k = 320.N/m.

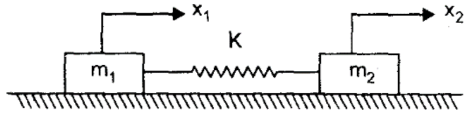


Fig. 5.12

Ans. The equations of motion can be written as

$$m_1 \ddot{x}_1 + k(x_1 - x_2) = 0 \,;\, m_2 \ddot{x}_2 + k(x_2 - x_1) = 0$$

Assuming the solution of the form

$$x_1 = A_1 \sin \omega t \,;\, x_2 = A_2 \sin \omega t$$

$$-m_1\omega^2 A_1 + k(A_1 - A_2) = 0$$

$$-m_2\omega^2 A_2 + k(A_2 - A_1) = 0$$

Amplitude ratio, $\frac{A_1}{A_2} = \frac{k}{k - m_1 w^2}$

$$\frac{A_1}{A_2} = \frac{k - m_2\omega^2}{k}$$

The frequency equation is obtained as

$$\frac{k}{k - m_1\omega^2} = \frac{k - m_2\omega^2}{k}$$

$$\omega^4 - \omega^2 k \frac{(m_1 + m_2)}{m_1 m_2} = 0$$

$$\omega^2 - \frac{k(m_1 + m_2)}{m_1 m_2} = 0$$

$$\omega_1 = 0$$

and $\omega_2 = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} = \sqrt{\frac{320(10+15)}{10 \times 15}} = 7.30 \text{ rad/sec.}$

$$\left(\frac{A_1}{A_2}\right)_{\omega_1} = 1.0$$

$$\left(\frac{A_1}{A_2}\right)_{\omega_2} = \frac{320 - 15(7.30)^2}{320} = -1.49$$

Q. 12. A vibratory system performs the motions as expressed by the following equations:

$$\ddot{x} + 800\,x + 90\,\theta = 0$$
$$\ddot{\theta} + 800\,\theta + 90\,x = 0$$

If the system is turned through 1.5 radians and released, find the frequencies and mode shapes

Ans. Adding both the equations, we get

$$\left(\ddot{x}+\ddot{\theta}\right) + (800 + 90)\,(x + \theta) = 0$$

Assuming $y_1 = x + \theta$ and substituting it in the above equation, we get

$$\ddot{y}_1 + 890\,y_1 = 0$$

From this equation, the frequency can be determined as

$$\omega_1 = \sqrt{890} = 29.83 \text{ rad/sec.}$$

Subtraction of the given equations is written as

$$\left(\ddot{x}-\ddot{\theta}\right) + (800 - 90)\,(x - \theta) = 0$$

Let us assume $y_2 = x - \theta$ and putting in the above equation

$$\ddot{y}_2 + 710\,y_2 = 0$$

Thus frequency, $\omega_2 = \sqrt{710} = 26.64$ rad/sec.

Assuming the motion to be harmonic type as

$$x = x_0 \sin \omega t$$
$$\theta = \theta_0 \sin \omega t$$
$$\ddot{x} = \omega^2 x_0 \sin \omega t$$
$$\theta = -\,\omega^2 \theta_0 \sin \omega t$$

Again rewriting the given equations and substituting the values of and 0

$$-\omega^2 x_0 + 800 x_0 + 90\theta = 0$$

$$(-\omega^2 + 800)\, x_0 = 90\,\theta_0$$

$$\left(\frac{x_0}{\theta_0}\right)_1 = \frac{-90}{-\omega_1^2 + 800} = \frac{-90}{-890+800} = 1$$

(substituting $\omega_1^2 = 890$)

and $\theta_0 = 1.5$ (given)

So $(x_0)_1 = 1.5 \times 1 = 1.5$ (first mode)

Similarly, for the second mode shape

$$\omega^2\theta_0 + 800\,\theta_0 + 90\, x_0 = 0$$

$$(-\omega^2 + 800)\,\theta_0 = -90\, x_0$$

$$\left(\frac{x_0}{\theta_0}\right)_2 = \frac{-\omega_2^2 + 800}{-90} = \frac{-710+800}{-90} = -1$$

So $(x_0)_2 = -1.5$

Q. 13. A machine runs at 5000 rpm. Its forcing frequency is very near to its natural frequency. If the nearest frequency of the machine is at least 20% from the forced frequency, design a suitable vibration absorber for the system. Assume the mass of the machine as 30 kg.

Ans. The natural frequency of the system at 5000 rpm.

$$\omega_n = \frac{2\pi N}{60} = \frac{2\pi \times 5000}{60} = 523.33 \text{ rad/sec.}$$

Assuming w = we can find two resonant frequencies from equation:

$$\left(\frac{\omega}{\omega_2}\right)^2 = \left(1+\frac{\mu}{2}\right) \pm \sqrt{\mu + \frac{\mu^2}{4}}$$

The resonant frequencies are at least 20% away from the forced frequency of the main system. So, we have

$$\omega/\omega_2 = 0.80$$

or $$\omega/\omega_2 = 1.20$$

When $\omega/\omega_2 = 0.8$, the value of μ

$$(0.8)^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(\mu+\frac{\mu^2}{4}\right)}$$

$$\mu = 0.2$$

and for $\omega/\omega_2 = 1.2$ the value of

$$\mu = 0.13$$

The larger value of μ is taken for design purpose.

$$\mu = 0.2 = \frac{m}{M} = \text{mass ratio}$$

$$m = 0.2 \times 30 = 6.0 \text{ kg}$$

$$\omega_1 = \sqrt{\frac{k_1}{M}} \qquad [\omega_1 = \omega_2]$$

$$\omega_n^2 = \frac{k_1}{M}$$

$$k_1 = \omega_n^2 \, M = (523.33)^2 \times 30 = 8216.22 \text{ KN/m}$$

$$\omega_2 = \sqrt{\frac{k_2}{m}} \; ; \; \omega_2 = \omega_n$$

$$k_2 = \omega_2^2 \, m = (523.33)^2 \times 6 = 1643.24 \text{ KN/m}.$$

Q. 14. Find the frequencies of the system shown in Fig. 5.13.

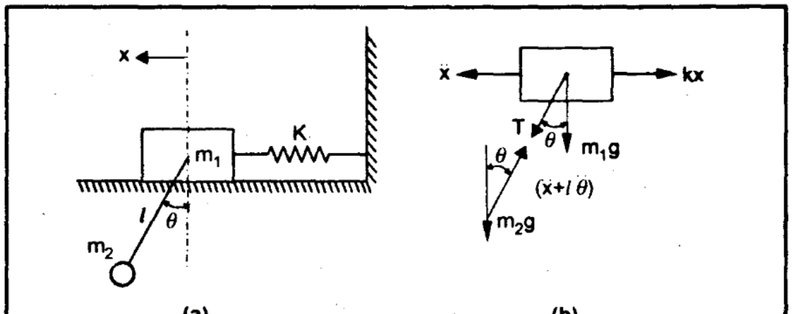


Fig. 5.13

Given : $m_1 = 2$ kg, $m_2 = 0.5$ kg
$K = 90$ N/m, $l = 0.25$ m.

Ans. Initially the pendulum rod is vertical and it is displaced by an angle 0 as shown in figure (a) and free body diagram of forces is shown in figure (b). Let us assume that T is the tension in the pendulum rod.

Resolving the forces vertically for m_2

$$m_2 g = T\cos\theta$$

Resolving the forces horizontally, T sin 0 will be known as restoring force as it downwards and brings m_2 to its original state. Horizontal displacement of m_2 is $x + l\sin\theta$.

when θ is very small, $\sin\theta \simeq \theta$ and $\cos\theta = 1$.

So horizontal displacement $= x + l\theta$

and acceleration $= \ddot{x} + l\ddot{\theta}$

Horizontal force $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

So $m_2 g = T$ and $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

or $m_2(\ddot{x} + l\ddot{\theta}) + T\theta = 0$

$$m_2(\ddot{x} + l\ddot{\theta}) + m_2 g\theta = 0, \text{ put } T = m_2 g$$

$$(\ddot{x} + l\ddot{\theta}) + g\ddot{\theta} = 0$$

or $l\ddot{\theta} + g\theta = -\ddot{x}$

Consider forces for mass rn1. All the forces are acting horizontally,

$$m_1\ddot{x} = -kx + T\sin\theta$$
$$= -kx + T\theta$$
$$m_1\ddot{x} + kx - T\theta = 0$$

Putting $T = m_2 g$

$$m_1\ddot{x} + kx - m_2 g\theta = 0$$
$$m_1\ddot{x} + kx = m_2 g\theta$$

Let us assume the solution of the form

$$x = A\sin\omega t$$

and $\theta = \phi\sin\omega t$

Substituting these solutions in the above two equations, we get

$$-l\omega^2\phi + g\phi - \omega^2 A = 0$$

and $$-m_1\omega^2 A + kA - m_2 g\phi = 0$$

$$\frac{A}{\phi} = \frac{-l\omega^2 + g}{\omega^2} = \frac{m_2 g}{k - m_1\omega^2}$$

The frequency equation can be written as

$$(-l\omega^2 + g)(k - m_1\omega^2) - \omega^2 m_2 g = 0$$

$$-kl\omega^2 + m_1 l\omega^4 + gk - m_1 g\omega^2 - \omega^2 m_2 g = 0$$

$$\omega^4 - \frac{(kl + m_1 g + m_2 g)\omega^2}{m_1 l} + \frac{gk}{m_1 l} = 0$$

So, $$\omega^2 = \frac{(m_1 + m_2)g + kl \pm \sqrt{[(m_1 + m_2)g + kl]^2 - 4m_1 lkg}}{2m_1 l}$$

Substituting the numerical values in the above equation

$$\omega^2 = \frac{(2 + 0.50)\,9.81 + 90 \times 0.25 \pm \sqrt{[(2 + 0.5)9.81 + 90 \times 2.5]^2 - 4 \times 2 \times 0.25 \times 90 \times 9.81}}{2 \times 2 \times 0.25}$$

$$= 24.5 + 22.5 \pm \sqrt{(24.5 + 22.5)^2 - 1764}$$

$$= 47 \pm \sqrt{2209 + 1764} = 47 \pm 21.095$$

$$\omega_1 = 8.25 \text{ rad/sec}, \omega_2 = 5.08 \text{ rad/sec.}$$

Q. 15. Find the natural frequencies of the there is no slip between cord and cylinder. system shown in Fig. 5.14. Assume that

Given :

k_1 **= 40 N/m**

k_2 **= 60 N/m**

m_1 **= 2 kg**

m_2 **= 10 kg**

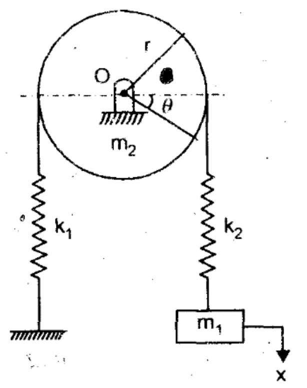


Fig. 5.14

Ans. Let us give x vertical displacement to mass as shown. Since there is no slip between the cord and cylinder, so vertical displacement x causes the cylinder to rotate by angle 0.
Writing the equations

$$m_1 \ddot{x} = -k_2 (x - r\theta)$$

and
$$I\ddot{\theta} = k_2(x - r\theta)\, r - k_1 r^2\theta$$

where $I = \frac{1}{2} m_2 r^2$ = moment of inertia of cylinder

Above equation becomes

$$m_1 \ddot{x} + k_2 x - k_2 r\theta = 0$$

$$I\ddot{\theta} + (k_1 r^2 + k_2 r^2)\theta - k_2 x r = 0$$

Let us assume the solution of the form

$$x = A \sin \omega t, \ \ddot{x} = -\omega^2 A \sin \omega t$$

$$\theta = \phi \sin \omega t, \ \ddot{\theta} = -\omega^2 \phi \sin \omega t$$

Substituting these values in the above equations

$$-\omega^2 A m_1 + k_2 A - k_2 r\phi = 0$$
$$-\omega^2 I\phi + (k_1 r^2 + k_2 r^2)\phi - k_2 A r = 0$$

$$(k_2 - \omega^2 m_1) A - k_2 r\phi = 0, \quad \frac{A}{\phi} = \frac{k_2 r}{k_2 - \omega^2 m_1}$$

$$(k_1 r^2 + k_2 r^2 - \omega^2 I)\phi - k_2 r A = 0, \quad \frac{A}{\phi} = \frac{k_1 r^2 + k_2 r^2 - \omega^2 I}{k_2 r}$$

$$-k_2^2 r^2 + (k_2 - \omega^2 m_1)(k_1 r^2 + k_2 r^2 - \omega^2 I) = 0$$

Also $I = 1/2\, m_2 r^2$

$$-k_2^2 r^2 + k_1 k_2 r^2 + k_2^2 r^2 - k_2 \frac{1}{2} m_2 r^2 \omega^2 - \omega^2 k_1 m_1 r^2 - \omega^2 m_1 k_2 r^2 + \omega^2 m_1 k\omega^2 \frac{1}{2} m_2 r^2 = 0$$

$$\omega^4 \frac{m_1 m_2 r^2}{2} - \omega^2\left(\frac{k_2 m_2 r^2}{2} + k_1 m_1 r^2 + m_1 k_2 r^2\right) + k_1 k_2 r^2 = 0$$

or $$\omega^4 - \omega^2\left(\frac{k_2 m_2 r^2}{m_1 m_2 r^2} + \frac{2k_1 m_1 r^2}{m_1 m_2 r^2} + \frac{2m_1 k_2 r^2}{m_1 m_2 r^2}\right) + \frac{2k_1 k_2 r^2}{m_1 m_2 r^2} = 0$$

$$\omega^4 - \omega^2\left(\frac{k_2}{m_1} + \frac{2k_1}{m_2} + \frac{2k_2}{m_2}\right) + \frac{2k_1 k_2}{m_1 m_2} = 0$$

$$\omega^4 - \omega^2\left[\frac{2(k_1 + k_2)}{m_2} + \frac{k_2}{m_1}\right] + \frac{2k_1 k_2}{m_1 m_2} = 0$$

Substituting the values of various parameters

$$\omega^4 - \omega^2\left[\frac{2(40+60)}{10} + \frac{60}{2}\right] + \frac{2\times 40\times 60}{2\times 10} = 0$$
$$\omega^4 - \omega^2(20 + 30) + 240 = 0$$
$$\omega^4 - 50\omega^2 + 240 = 0$$
$$\omega^2 = \frac{50 \pm \sqrt{2500 - 960}}{2} = \frac{50 \pm 39.24}{2}$$
$$\omega_1 = \sqrt{44.62}\ \text{rad/sec} = 6.68\ \text{rad/sec}$$
$$\omega_2 = \sqrt{5.38}\ \text{rad/sec} = 2.32\ \text{rad/sec}$$

Q. 16. Two bodies having equal masses as 60 kg each and radius of gyration 0.3 m are keyed to both ends of a shaft 0.80 m long. The shaft is 0.08 m in diameter for 0.30 m length, 0.10 diameter for 0.20 m length and 0.09 m diameter for rest of the length. Find the frequency of torsional vibrations.

Take G = 9×10^{11} N/m^2

Ans.

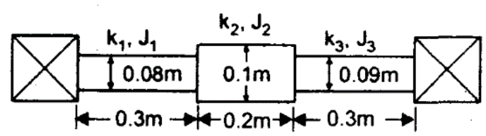


Fig. 5.15

$$I = mk^2 = 60 \times .3 \times .3 = 5.4 \text{ kg m}^2$$

$$k_1 = \frac{GJ_1}{l_1} = \frac{9\times10^{11}\times(\pi/32)\times(0.8)^4}{.30}$$

$$= 1.2057 \times 10^7 \text{ N} - m/\text{rad}$$

$$k_2 = \frac{GJ_2}{l_2} = \frac{9\times10^{11}\times(\pi/32)\times(0.10)^4}{.2}$$

$$= 4.415 \times 10^7 \text{ N-m/rad}$$

$$k_3 = \frac{GJ_3}{l_3} = \frac{9\times10^{11}\times(\pi/32)\times(.09)^4}{.3}$$

$$= 1.93139 \times 10^7 \text{ N-m/rad}$$

where I = mass moment of inertia, J_1, J_2 and J_3 are polar moment of inertia.

The equivalent stiffness of the shaft is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Parts of shaft are connected in series

$$= \left(\frac{1}{1.2057} + \frac{1}{4.415} + \frac{1}{1.93139}\right)10^{-7}$$

$$= (0.829 + .2265 + .51776)10^{-7}$$

$$k = 6.356 \times 10^{6} \text{ N-m/rad}$$

$$\omega = \sqrt{\frac{k(I_1 + I_2)}{I_1 I_2}} = \sqrt{\frac{2k}{I}} \qquad \text{if } I_1 = I_2$$

$$\omega = \left(\frac{2 \times 6.356 \times 10^{6}}{5.4}\right)^{\frac{1}{2}} = 1.534 \times 10^{3} \text{ rad/sec.}$$

Q. 17. Two equal masses of weight 400 N each and radius of gyration 40 cm are keyed to the opposite end of a shaft 60 cm long. The shaft is 7.5 cm diameter for the first 25 cm of its length, 12.5 cm diameter for next 10 cm and 8.5 cm diameter for the remaining length. Find the frequency of free torsional vibrations of the system and position of node. Take G = 0.84 106 kg/cm2

Ans. The system is shown in figure

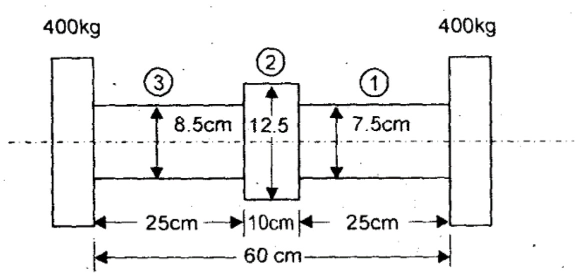


Fig. 5.16

$$k_{t_1} = \left(\frac{GJ}{l}\right)_1 = \frac{0.84 \times 10^6}{25} \times \pi/32 \times (7.5)^4$$

$$= 10.43 \times 10^6 \text{ kg–cm/rad}$$

$$k_{t_2} = \frac{0.84 \times 10^6}{10} \times \pi/32 \times (12.5)^4$$

$$= 10^6 \times 201.2 \text{ kg –cm/rad}$$

$$k_{t_3} = \frac{0.84 \times 10^6}{25} \times \pi/32 \times (8.5)^4 = 17.2 \times 10^6 \text{ kg –cm/rad}$$

k_{t_1}, k_{t_2} and k_{t_3} are connected in series, so equivalent stiffiness of shaft

$$\frac{1}{k_{te}} = \frac{1}{k_{t_1}} + \frac{1}{k_{t_2}} + \frac{1}{k_{t_3}}$$

$$\frac{1}{k_{te}} = 10^{-6}\left(\frac{1}{10.43} + \frac{1}{201.2} + \frac{1}{17.2}\right)$$

$$k_{te} = 6.29 \times 10^6 \text{ kg-cm/rad} = 6.29 \times 10^4 \text{ kg.m/rad}$$

The expression for frequency is

$$\omega_n = \sqrt{\frac{k_t(I_1 + I_2)}{I_1 I_2}}$$

For the system, $I_1 = I_2 = I$ (say)

So $I = \frac{w}{g}k^2 = \frac{400}{9.81} \times (40)^2 = 652.4 \times 10^2$ kg-cm^2

$$\omega_n = \sqrt{K_t \frac{2I}{I^2}}$$

$$\omega_n = \sqrt{\frac{2K_t}{I}} = \sqrt{\frac{2 \times 6.29 \times 10^4}{6.524}} = 138.86 \text{ rad/sec.}$$

Since $I_1 = I_2$, so the node will lie in the middle of the equivalent shaft.
Let us find the length of equivalent shaft

$$l_e = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4$$

$$= 25 + 10 \left(\frac{7.5}{12.5}\right)^4 + 25 \left(\frac{7.5}{8.5}\right)^4$$

$$= 41.45 \text{ cm}$$

The middle of equivalent shaft is 20.72 cm from the left hand side.

Q. 18. Find the natural frequency of the system shown in Fig 5.17.

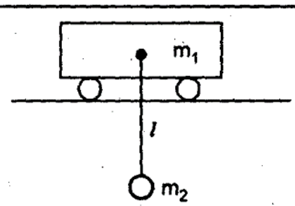


Fig. 5.17

Ans. Let us assume the whole system is moved to the right by x. The ball m_2 is displaced by θ as shown in figure 5.18. The total movement of ball m_2 is $x + l\theta$.

The equations of motion are

For pendulum,

$$m_2(\ddot{x}+l\ddot{\theta}) = -T\theta \qquad (\sin\theta = \theta)$$

$$(T = m_2 g)$$

$$m_2(\ddot{x}+l\ddot{\theta}) + m_2 g\theta = 0$$

$$\ddot{x} + l\ddot{\theta} + g\theta = 0 \qquad ...(1)$$

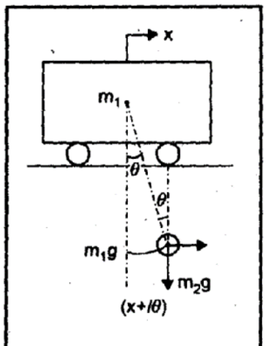


Fig. 5.18

For mass m_1,

$$m_1\ddot{x} = T\theta$$

$$m_1\ddot{x} - m_2 g\,\theta = 0$$

or

$$\ddot{x} = \frac{m_2}{m_1}g\theta \qquad ...(2)$$

Putting the value of $\ddot{x}$ from equation (2) in equation (1), we get

$$\frac{m_2}{m_1}g\theta + l\ddot{\theta} + g\theta = 0$$

$$l\ddot{\theta} + \left(g+\frac{m_2 g}{m_1}\right)\theta = 0$$

$$\ddot{\theta}+\left(\frac{g}{l}+\frac{m_2 g}{m_1 l}\right)\theta = 0$$

$$\ddot{\theta}+\frac{g}{m_1 l}(m_1+m_2)\theta = 0$$

So

$$\omega_n = \sqrt{\frac{g}{m_1 l}(m_1+m_2)}$$

Q. 19. Derive the natural frequencies of the system shown in Fig. 5.19.

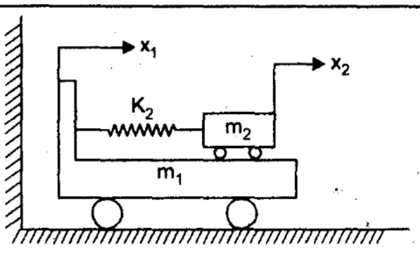


Fig. 5.19

Given $m_1 = 196$ kg, $k_1 = 98000$ N/m
$m_2 = 49$ kg, $k_2 = 19600$ N/m

Ans. The equations of motion for the system shown in figure can be written as

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = 0$$
$$m_2 \ddot{x}_2 - k_2 (x_1 - x_2) = 0$$

Rearranging the above equations, we can write them as

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 x_1 - k_2 x_2 = 0$$
$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0$$

Let us assume the solution of the form

$$x_1 = A_1 \sin \omega t$$

and $$x_2 = A_2 \sin \omega t$$

So $$\ddot{x}_1 = -\omega^2 A_1 \sin \omega t$$

$$\ddot{x}_2 = -\omega^2 A_2 \sin \omega t$$

The above equations can be written as

$$-m_1 \omega^2 A_1 + k_1 A_1 + k_2 A_1 - k_2 A_2 = 0$$
$$-m_2 \omega^2 A_2 + k_2 A_2 - k_2 A_1 = 0$$

or $$(-m_1 \omega^2 + k_1 + k_2)(-m_2 \omega^2 + k_2) - k_2^2 = 0$$

$$\omega^4 m_1 m_2 - m_1 k_2 \omega^2 - k_1 m_2 \omega^2 + k_1 k_2 - k_2 m_2 \omega^2 = 0$$

So $\omega^4 - \omega^2\left(\frac{k_2}{m_2}+\frac{k_1}{m_1}+\frac{k_2}{m_1}\right)+\frac{k_1 k_2}{m_1 m_2} = 0$

Substituting the values of various terms, we get

$$\omega^4 - 1000\omega^2 + 2 \times 10^5 = 0$$

Thus $\omega_1 = 26.9$ rad/sec. ; $\omega_2 = 16.6$ rad/sec.

Q. 20. Two rotors A and B are attached to the end of a shaft 50 cm long. Weight of rotor A is 300 N and its radius of gyration is 30 cm and the corresponding values of B are 500 N and 45 cm respecitvely. The shaft is 7cm in diameter for first 25 cm, 12 cm in diameter for next 10 cm and 10 cm diameter for remaining length. Modulus of rigidity for shaft material is 8 x 106 kg/cm2 Find:

(i)the position of node and

(ii) the frequency of torsional vibrations.

Ans. The configuration diagram is shown in

$$W_A = 300\text{ N}, W_B = 500\text{ N}$$

or $$m_A = \frac{W_A}{g} = \frac{300}{9.81} kg,$$

and $$m_B = \frac{W_B}{g} = \frac{500}{9.81} kg,$$

$$K_A = 30\text{ cm}, K_B = 45\text{ cm}$$

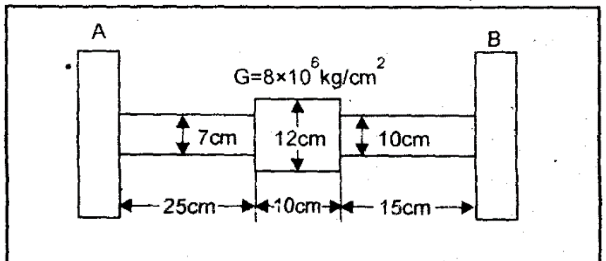


Fig. 5.20

The shaft may be converted into a torsionally equivalent shaft the length of which is given by (Assuming d = 7 cm)

$$l = l_1 + l_2\left(\frac{d}{d_2}\right)^4 + l_3\left(\frac{d}{d_3}\right)^4 = 29.76 \text{ cm}$$

Let N be the position of node for the two rotor system and the length of two parts of equivalent shaft be 1A and 1B as shown in Fig. 5,21.

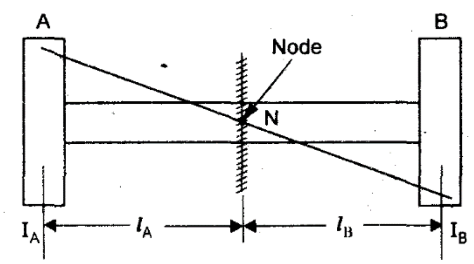


Fig. 5.21

So, $$l = l_A + l_B \qquad ...(i)$$

We know that,

$$\omega_A = \omega_B = \sqrt{\frac{K_{tA}}{I_A}} = \sqrt{\frac{K_{tB}}{I_B}}$$

$$K_t = \frac{T}{\theta} = \frac{GJ}{l}$$

(where J = Polar Moment of inertia of shaft)

$$J = \pi/32 d^4$$

or $$\frac{G}{I_A l_A}\left(\frac{\pi}{32}d^4\right) = \frac{G}{I_B l_B}\left(\frac{\pi}{32}d^4\right)$$

$$\frac{l_A}{l_B} = \frac{I_B}{I_A} = \frac{m_B K_B^2}{m_A K_A^2} = \frac{500(0.45)^2 \times 9.81}{9.81 \times 300 \times (.30)^2}$$

or $$\frac{l_A}{l_B} = 3.75 \qquad ...(ii)$$

From equations (i) and (ii), we get

$$l_A = 23.5 \text{ cm}, \; l_B = 6.26 \text{ cm}$$

Now $$\omega_B = \sqrt{\frac{G}{I_B l_B} \cdot \frac{\pi}{32} d^4} = \sqrt{\frac{8 \times 10^6}{\frac{500}{981}(45)^2 \times 6.26} \times \frac{\pi}{32}(7)^4}$$

$$= 540.24 \text{ rad/sec.}$$

Since $$\omega_B = \omega_A = 540.24 \text{ rad/sec.}$$

or $$f_B = f_A = \frac{540.24}{2\pi} = 85.98 \text{ cycles/sec.}$$

Q. 21. what is a two degree system?

Ans. In a two degree freedom system, any point in the system may execute harmonic of the two natural frequencies and these are known of vibration. Let us assume the motion of two masses is simple harmonic and is represented as

$$x_1 = X_1 \sin \omega t$$
$$x_2 = X_2 \sin \omega t$$

where X1 and X2are the amplitudes of two masses respectively and are referred as principal co-ordinal.

Q. 22. Derive the equation of motion of the system shown in figure below and find its frequencies.

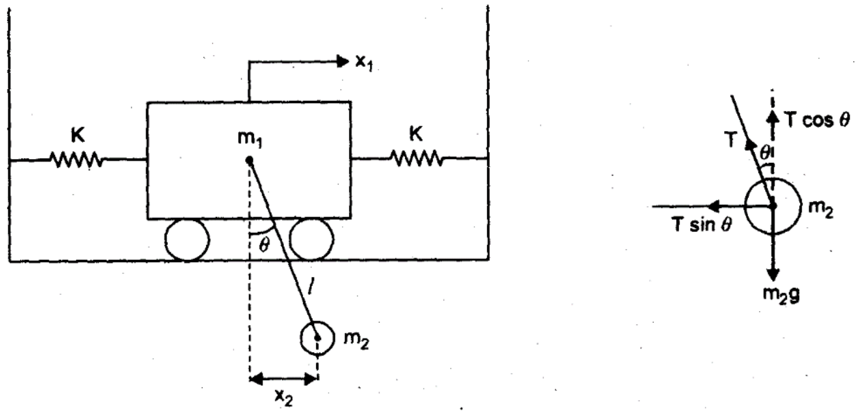


Fig. 5.22

Ans.

$$m_1 \ddot{x}_1 = -2\,Kx + T \sin\theta$$

$$m_1 \ddot{x}_1 + 2Kx - T \sin\theta = 0$$

$$m_1 \ddot{x}_1 + 2Kx - T\,\theta = 0 \qquad \text{...(I)}$$

Now $m_2 g = T\cos\theta$

When θ is small, $\cos\theta = 1$

$$m_2 g = T$$

Putting in equation (I), we get

$$m_1 \ddot{x}_1 + 2Kx - m_2 g\theta = 0 \qquad \text{...(II)}$$

$$m_2 \ddot{x}_2 = -T\sin\theta$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) = -T\theta \qquad [x_2 = x_1 + l\theta]$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) + T\,\theta = 0$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) + m_2 g\,\theta = 0$$

$$l\,\ddot{\theta} + g\,\theta = -\ddot{x}_1 \qquad \text{...(III)}$$

$$\omega^2 = \frac{\dfrac{m_1 g + m_2 g + 2Kl}{m_1 l} \pm \sqrt{\left(\dfrac{m_1 g + m_2 g + 2Kl}{m_1 l}\right)^2 - \dfrac{4 \times 2\,Kg}{m_1 l}}}{2}$$

$$\omega^2 = \frac{m_1 g + m_2 g + 2Kl \pm \sqrt{(m_1 g + m_2 g + 2Kl)^2 - 8\,Kg\,m_1 l}}{2m_1 l}$$

$$\omega = \left[\frac{m_1 g + m_2 g + 2Kl \pm \sqrt{(m_1 g + m_2 g + 2Kl)^2 - 8\,Kg\,m_1 l}}{2m_1 l}\right]^{1/2}$$

Let $x_1 = A \sin \omega t \; ; \; \ddot{x}_1 = -A\,\omega^2 \sin \omega t$

$\theta = \phi \sin \omega t \; ; \; \ddot{\theta} = -\phi\,\omega^2 \sin \omega t$

Equation (II) becomes

$$-m_1 \omega^2 A + 2KA - m_2 g \phi = 0$$

$$A\,(2K - m_1 \omega^2) - (m_2 g)\,\phi = 0$$

$$\frac{A}{\phi} = \frac{m_2 g}{2K - m_1 \omega^2} \qquad \text{...}(i)$$

Equation (III) becomes

$$-\,l\,\phi\,\omega^2 \sin \omega t + g\,\phi \sin \omega t = A\,\omega^2 \sin \omega t$$

$$\phi\,(g - l\omega^2) = A\omega^2$$

$$\frac{A}{\phi} = \frac{g - l\omega^2}{\omega^2} \qquad \text{...}(ii)$$

$$\frac{A}{\phi} = \frac{m_2 g}{2K - m_1 \omega^2} = \frac{g - l\omega^2}{\omega^2}$$

The frequency equation can be written as

$$m_2 g\omega^2 = 2Kg - 2Kl\omega^2 - m_1 g\omega^2 + m_1 l\omega^4$$

$$m_1 l\omega^4 - (m_1 + m_2) g\omega^2 - 2Kl\omega^2 + 2Kg = 0$$

$$\omega^4 - \frac{[m_1 g + m_2 g + 2Kl]\,\omega^2}{m_1 l} + \frac{2Kg}{m_1 l} = 0$$

Q. 23. What are various methods available for vibration control?

Ans. Various methods of vibration control are

1. Vibration Absorbers (centrifugal pendulum absorber, Lanchester damper, Houdaille damper).
2. Vibration Isolation materials like rubber, cork, felt, pad etc.

Q. 24. Calculate the natural frequency of a shaft of diameter 10 cm and length 300 cm carrying two discs of diameters 125 cm and 200 cm respectively at its ends and weighing 480 N and 900 N respectively. Modulus of the rigidity of the shaft may be taken as 2 x 1011 N/m2.

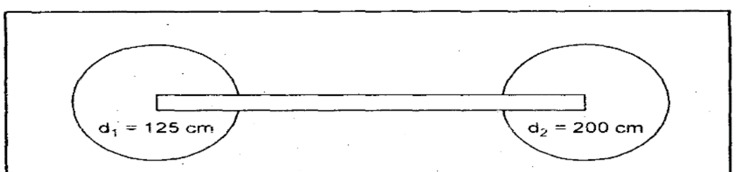


Fig. 5.23

l = 300 cm = 3 m
d = 10 cm = 0.1 m
w_1 = 480 N
w_2 = 900 N
$C = 2 \times 10^{11}$ N/m^2

$$I_1 = \frac{w_1}{g}\frac{r_1^2}{2} = \frac{480}{9.81} \times \left(\frac{62.5}{100}\right)^2 \times \frac{1}{2} = 9.56 \text{ kg.m}^2$$

$$I_2 = \frac{w_2}{g}\frac{r_2^2}{2} = \frac{900}{9.81} \times \left(\frac{100}{100}\right)^2 \times \frac{1}{2} = 45.87 \text{ kg.m}^2$$

$$K_T = \frac{CJ}{l}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32} (.10)^4 = 9.81 \times 10^{-6} \text{ m}^4$$

$$l = 3 \text{ m}$$

$$K_T = \frac{2 \times 10^{11} \times 9.81 \times 10^{-6}}{3}$$

$$K_T = 6.54 \times 10^5 \text{ Nm/rad}$$

$$w_n = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s}$$

$$= \sqrt{\frac{6.54 \times 10^5 (9.56 + 45.87)}{9.56 \times 45.87}}$$

$$w_n = 287.52 \text{ rad/s}$$

$$f_n = \frac{w_n}{2\pi} = \frac{287.52}{2\pi} = 45.78 \text{ Hz}$$

Q. 25. What is the difference between a vibration absorber and a vibration isolator?

Ans. Vibration absorber is a s ring mass system attached to the main vibratory system to absorb the vibrations whereas vibration isolators are material like cork , rubber, felt, pad etc. which are used to isolate machines from its foundation and support. These vibration isolation absorb the shocks forces set up in the machinery and prevents the damage of foundations and supports.

Department of Aeronautical Engineering

U20AE602 VIBRATION AND ELEMENTS OF AEROELASTICITY

QUESTION BANK
PART B

1. **Draw the mode shapes for two rotor system.**

Ans.

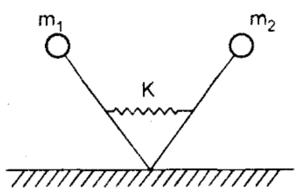


Fig. 5.1

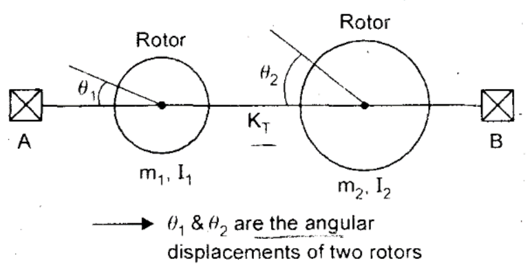


Fig. 5.2

Torsional Vibrations : Consider fig. 5.2. A shaft AB is carrying two rotors of moment of inertia I and '2 Let and °2 be the angular displacements of rotor at any .instant from mean position. The equation of motion can be written s, any .instant from mean position. The equation of motion can be written s,

$$I_1 \ddot{\theta}_1 = -K_T(\theta_1 - \theta_2) \quad \text{...(I)}$$

$$I_1 \ddot{\theta}_1 + K_T(\theta_1 - \theta_2) = 0 \quad \text{...(II)} \quad \left[\ddot{\theta}_1 = \frac{d^2\theta_1}{dt^2} \right]$$

$$I_2 \ddot{\theta}_2 = -K_T(\theta_2 - \theta_1) \quad \text{...(III)}$$

$$I_2 \ddot{\theta}_2 + K_T(\theta_2 - \theta_1) = 0 \quad \text{...(IV)} \quad \left[\ddot{\theta}_2 = \frac{d^2\theta_2}{dt^2} \right]$$

Put,

$$\theta_1 = a_1 \sin \omega t, \theta_2 = a_2 \sin \omega t$$

$$\ddot{\theta}_1 = -a_1 \omega^2 \sin \omega t$$

$$\ddot{\theta}_1 = -\omega^2 \theta_1, \text{ similarly, } \ddot{\theta}_2 = -\omega^2 \theta_2$$

Putting these values in (II) and (IV)

$$I_1 \times -\omega^2 \theta_1 + K_T (a_1 - a_2) \sin \omega t = 0$$

$$-I_1 \omega^2 a_1 \sin \omega t + K_T (a_1 - a_2) \sin \omega t = 0$$

$$-\omega^2 I_1 a_1 + K_T (a_1 - a_2) = 0 \quad \text{...(V)}$$

$$I_2 \times -\omega^2 \theta_2 + K_T (a_2 - a_1) \sin \omega t = 0$$

$$I_2 \times -\omega^2 \times a_2 \sin \omega t + K_T (a_2 - a_1) \sin \omega t = 0$$

$$I_2 \omega^2 a_2 + K_T (a_2 - a_1) = 0 \quad \text{...(VI)}$$

$$\omega^2 I_1 a_1 + K_T (a_1 - a_2) = 0$$

$$\omega^2 I_2 a_2 + K_T (a_2 - a_1) = 0$$

$$(K_T - I_1\omega^2) a_1 - K_T a_2 = 0$$

$$-K_T a_1 + (K_T - I_2\omega^2) a_2 = 0$$

Solving by determinant

$$\begin{vmatrix} K_T - I_1\omega^2 & -K_T \\ -K_T & K_T - I_2\omega^2 \end{vmatrix} = 0$$

$$(K_T - I_1 \omega^2)(K_T - I_2\omega^2) - K_T^2 = 0$$

$$K_T^2 - K_T I_2 \omega^2 - K_T I_1 \omega^2 + I_1 I_2 \omega^4 - K_T^2 = 0$$

$$\omega^2 (I_1 I_2 \omega^2 - K_T I_1 - K_T I_2) = 0$$

$$\omega^2 = 0$$

$$\boxed{\omega_1 = 0} \quad \text{...(1)}$$

$$I_1 I_2 \omega^2 - K_T (I_1 + I_2) = 0$$

$$\omega_2 = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s} \quad \text{...(2)}$$

Put value of $\omega_1 = 0$ in (VII)

$$a_1 K_T - K_T a_2 = 0$$

$$\boxed{\frac{a_1}{a_2} = 1} \quad \text{...(IX)}$$

From (VIII)

$$\frac{a_1}{a_2} = \frac{K_T - I_2\omega^2}{K_T}$$

$$\frac{a_1}{a_2} = 1 - \frac{I_2\omega^2}{K_T}$$

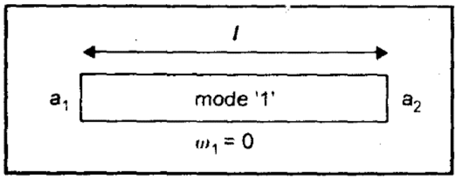
Put value of $\omega_2 = \sqrt{\frac{K_T(I_1 + I_2)}{I_1 I_2}}$ rad/s

$$\frac{a_1}{a_2} = 1 - \frac{I_2}{K_T} \times \frac{K_T(I_1 + I_2)}{I_1 I_2}$$

$$\frac{a_1}{a_2} = 1 - \left(1 + \frac{I_2}{I_1}\right) = \frac{-I_2}{I_1}$$

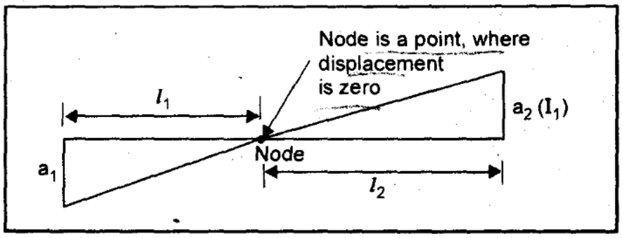
It shows that the angular displacements of rotors are inversely proportional to their inertia.
The section of the shaft where angular displacement is zero Is known as node. First Mode shape

$$\omega_1 = 0, \frac{a_1}{a_2} = 1$$



Second Mode Shape

$$\omega_2 = \sqrt{K_T \frac{(I_1 + I_2)}{I_1 I_2}}, \frac{a_1}{a_2} = \frac{-I_2}{I_1}$$



$$\omega = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s}$$

2. **Find the frequencies of the system shown in Fig. 5.13.**

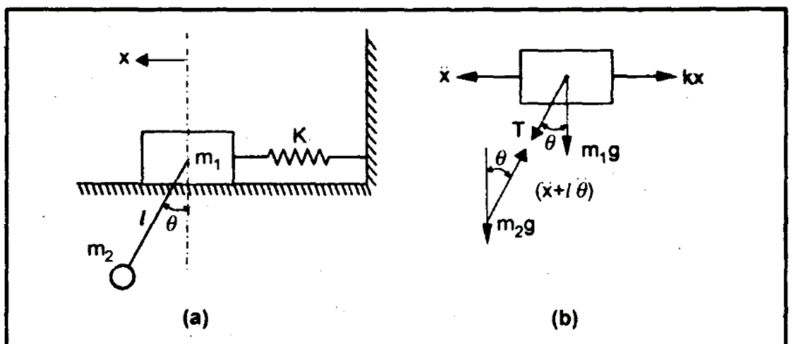


Fig. 5.13

Given : $m_1 = 2$ kg, $m_2 = 0.5$ kg
$K = 90$ N/m, $l = 0.25$ m.

Ans. Initially the pendulum rod is vertical and it is displaced by an angle 0 as shown in figure (a) and free body diagram of forces is shown in figure (b). Let us assume that T is the tension in the pendulum rod.Resolving the forces vertically for m_2

$$m_2 g = T \cos\theta$$

Resolving the forces horizontally, T sin 0 will be known as restoring force as it downwards and brings m_2 to its original state. Horizontal displacement of m_2 is $x + l \sin\theta$.

when θ is very small, $\sin\theta \simeq \theta$ and $\cos\theta = 1$.

So horizontal displacement $= x + l\theta$

and acceleration $= \ddot{x} + l\ddot{\theta}$

Horizontal force $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

So $m_2 g = T$ and $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

or $m_2(\ddot{x} + l\ddot{\theta}) + T\theta = 0$

$$m_2(\ddot{x} + l\ddot{\theta}) + m_2 g\,\theta = 0, \text{ put } T = m_2 g$$

$$(\ddot{x} + l\ddot{\theta}) + g\ddot{\theta} = 0$$

or $l\ddot{\theta} + g\theta = -\ddot{x}$

Consider forces for mass m1. All the forces are acting horizontally,

$$m_1\ddot{x} = -kx + T\sin\theta$$

$$= -kx + T\theta$$

$$m_1\ddot{x} + kx - T\theta = 0$$

Putting $T = m_2 g$

$$m_1\ddot{x} + kx - m_2 g\theta = 0$$

$$m_1\ddot{x} + kx = m_2 g\theta$$

Let us assume the solution of the form

$$x = A\sin\omega t$$

and $\theta = \phi\sin\omega t$

Substituting these solutions in the above two equations, we get

$$-l\omega^2\phi + g\phi - \omega^2 A = 0$$

and $-m_1\omega^2 A + kA - m_2 g\phi = 0$

$$\frac{A}{\phi} = \frac{-l\omega^2 + g}{\omega^2} = \frac{m_2 g}{k - m_1\omega^2}$$

The frequency equation can be written as

$$(-l\omega^2 + g)(k - m_1\omega^2) - \omega^2 m_2 g = 0$$

$$-kl\omega^2 + m_1 l\omega^4 + gk - m_1 g\omega^2 - \omega^2 m_2 g = 0$$

$$\omega^4 - \frac{(kl + m_1 g + m_2 g)\omega^2}{m_1 l} + \frac{gk}{m_1 l} = 0$$

So,

$$\omega^2 = \frac{(m_1 + m_2)g + kl \pm \sqrt{[(m_1 + m_2)g + kl]^2 - 4m_1 lkg}}{2m_1 l}$$

Substituting the numerical values in the above equation

$$\omega^2 = \frac{(2 + 0.50)\,9.81 + 90 \times 0.25 \pm \sqrt{[(2 + 0.5)9.81 + 90 \times 2.5]^2 - 4 \times 2 \times 0.25 \times 90 \times 9.81}}{2 \times 2 \times 0.25}$$

$$= 24.5 + 22.5 \pm \sqrt{(24.5 + 22.5)^2 - 1764}$$

$$= 47 \pm \sqrt{2209 + 1764} = 47 \pm 21.095$$

$$\omega_1 = 8.25 \text{ rad/sec}, \omega_2 = 5.08 \text{ rad/sec.}$$

---

3. **Find the natural frequencies of the there is no slip between cord and cylinder system shown in Fig. 5.14. Assume that**

Given :

$$k_1 = 40 \text{ N/m}$$
$$k_2 = 60 \text{ N/m}$$
$$m_1 = 2 \text{ kg}$$
$$m_2 = 10 \text{ kg}$$

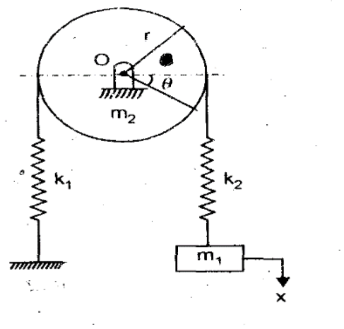


Fig. 5.14

Ans. Let us give x vertical displacement to mass as shown. Since there is no slip between the cord and cylinder, so vertical displacement x causes the cylinder to rotate by angle 0.

Writing the equations

$$m_1 \ddot{x} = -k_2 (x - r\theta)$$

and $$I\ddot{\theta} = k_2(x - r\theta)\, r - k_1 r^2\theta$$

where $I = \frac{1}{2}m_2r^2$ **= moment of inertia of cylinder**

Above equation becomes

$$m_1 \ddot{x} + k_2x - k_2r\theta = 0$$

$$I\ddot{\theta} + (k_1r^2 + k_2r^2)\theta - k_2xr = 0$$

Let us assume the solution of the form

$$x = A \sin \omega t,\ \ddot{x} = -\omega^2 A \sin \omega t$$

$$\theta = \phi \sin \omega t,\ \ddot{\theta} = -\omega^2 \phi \sin \omega t$$

Substituting these values in the above equations

$$-\omega^2 Am_1 + k_2A - k_2r\phi = 0$$

$$-\omega^2 I \phi + (k_1r^2 + k_2r^2)\phi - k_2Ar = 0$$

$$(k_2 - \omega^2 m_1) A - k_2r\phi = 0,\ \frac{A}{\phi} = \frac{k_2r}{k_2 - \omega^2 m_1}$$

$$(k_1r^2 + k_2r^2 - \omega^2 I)\phi - k_2rA = 0,\ \frac{A}{\phi} = \frac{k_1r^2 + k_2r^2 - \omega^2 I}{k_2r}$$

$$-k_2^2r^2 + (k_2 - \omega^2 m_1)(k_1r^2 + k_2r^2 - \omega^2 I) = 0$$

Also $I = 1/2\ m_2r^2$

$$-k_2^2r^2 + k_1 k_2 r^2 + k_2^2 r^2 - k_2 \frac{1}{2} m_2r^2\omega^2 - \omega^2 k_1m_1r^2 - \omega^2 m_1k_2r^2 + \omega^2 m_1 k\omega^2 \frac{1}{2} m_2r^2 = 0$$

$$\omega^4 \frac{m_1m_2r^2}{2} - \omega^2\left(\frac{k_2m_2r^2}{2} + k_1 m_1 r^2 + m_1 k_2 r^2\right) + k_1k_2r^2 = 0$$

or $$\omega^4 - \omega^2 \left(\frac{k_2m_2r^2}{m_1m_2r^2} + \frac{2k_1 m_1r^2}{m_1m_2r^2} + \frac{2m_1k_2r^2}{m_1m_2r^2}\right) + \frac{2k_1k_2r^2}{m_1m_2r^2} = 0$$

$$\omega^4 - \omega^2\left(\frac{k_2}{m_1}+\frac{2k_1}{m_2}+\frac{2k_2}{m_2}\right)+\frac{2k_1k_2}{m_1m_2} = 0$$

$$\omega^4 - \omega^2\left[\frac{2(k_1+k_2)}{m_2}+\frac{k_2}{m_1}\right] + \frac{2k_1k_2}{m_1m_2} = 0$$

Substituting the values of various parameters

$$\omega^4 - \omega^2\left[\frac{2(40+60)}{10}+\frac{60}{2}\right] + \frac{2\times40\times60}{2\times10} = 0$$

$$\omega^4 - \omega^2 (20 + 30) + 240 = 0$$

$$\omega^4 - 50\omega^2 + 240 = 0$$

$$\omega^2 = \frac{50\pm\sqrt{2500-960}}{2} = \frac{50\pm39.24}{2}$$

$$\omega_1 = \sqrt{44.62}\ \text{rad/sec} = 6.68\ \text{rad/sec}$$

$$\omega_2 = \sqrt{5.38}\ \text{rad/sec} = 2.32\ \text{rad/sec}$$

4. Find the natural frequency of the system shown in Fig 5.17.

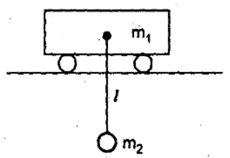


Fig. 5.17

Ans. Let us assume the whole system is moved to the right by x. The ball m_2 is displaced by θ as shown in figure 5.18. The total movement of ball m_2 is $x + l\theta$.

The equations of motion are

For pendulum,

$$m_2(\ddot{x}+l\ddot{\theta}) = -T\theta \qquad (\sin\theta = \theta)$$

$$(T = m_2 g)$$

$$m_2(\ddot{x}+l\ddot{\theta}) + m_2 g\theta = 0$$

$$\ddot{x} + l\ddot{\theta} + g\theta = 0 \qquad ...(1)$$

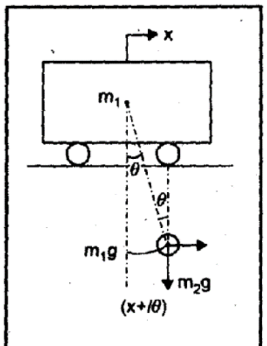


Fig. 5.18

For mass m_1,

$$m_1\ddot{x} = T\theta$$

$$m_1\ddot{x} - m_2 g\theta = 0$$

or

$$\ddot{x} = \frac{m_2}{m_1} g\theta \qquad ...(2)$$

Putting the value of $\ddot{x}$ from equation (2) in equation (1), we get

$$\frac{m_2}{m_1} g\theta + l\ddot{\theta} + g\theta = 0$$

$$l\ddot{\theta} + \left(g + \frac{m_2 g}{m_1}\right)\theta = 0$$

$$\ddot{\theta} + \left(\frac{g}{l} + \frac{m_2 g}{m_1 l}\right)\theta = 0$$

$$\ddot{\theta} + \frac{g}{m_1 l}(m_1 + m_2)\theta = 0$$

So

$$\omega_n = \sqrt{\frac{g}{m_1 l}(m_1 + m_2)}$$

---

5. **Two bodies having equal masses as 60 kg each and radius of gyration 0.3 m are keyed to both ends of a shaft 0.80 m long. The shaft is 0.08 m in diameter for 0.30 m length, 0.10 diameter for 0.20 m length and 0.09 m diameter for rest of the length. Find the frequency of torsional vibrations.**

Take G = 9 × 10^{11} N/m^2

Ans.

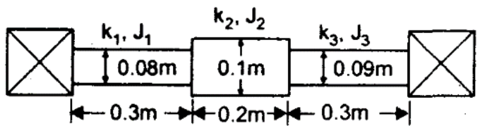


Fig. 5.15

$$I = mk^2 = 60 \times .3 \times .3 = 5.4 \text{ kg m}^2$$

$$k_1 = \frac{GJ_1}{l_1} = \frac{9\times10^{11}\times(\pi/32)\times(0.8)^4}{.30}$$

$$= 1.2057 \times 10^7 \text{ N} - m/\text{rad}$$

$$k_2 = \frac{GJ_2}{l_2} = \frac{9\times10^{11}\times(\pi/32)\times(0.10)^4}{.2}$$

$$= 4.415 \times 10^7 \text{ N-m/rad}$$

$$k_3 = \frac{GJ_3}{l_3} = \frac{9\times10^{11}\times(\pi/32)\times(.09)^4}{.3}$$

$$= 1.93139 \times 10^7 \text{ N-m/rad}$$

where I = mass moment of inertia, J_1, J_2 and J_3 are polar moment of inertia.

The equivalent stiffness of the shaft is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Parts of shaft are connected in series

$$= \left(\frac{1}{1.2057} + \frac{1}{4.415} + \frac{1}{1.93139}\right)10^{-7}$$

$$= (0.829 + .2265 + .51776)10^{-7}$$

$$k = 6.356 \times 10^6 \text{ N-m/rad}$$

$$\omega = \sqrt{\frac{k(I_1 + I_2)}{I_1 I_2}} = \sqrt{\frac{2k}{I}} \qquad \text{if } I_1 = I_2$$

$$\omega = \left(\frac{2\times6.356\times10^6}{5.4}\right)^{\frac{1}{2}} = 1.534 \times 10^3 \text{ rad/sec.}$$

---

6. **Derive the equation of motion of the system shown in figure below and find its frequencies.**

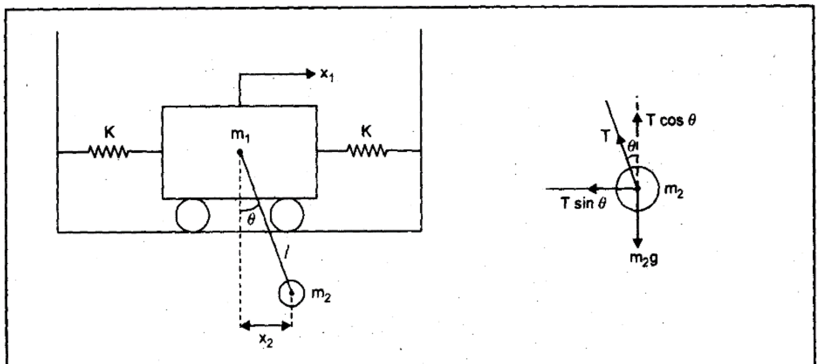


Fig. 5.22

Ans.

$$m_1 \ddot{x}_1 = -2\,Kx + T\sin\theta$$

$$m_1 \ddot{x}_1 + 2Kx - T\sin\theta = 0$$

$$m_1 \ddot{x}_1 + 2Kx - T\,\theta = 0 \qquad \text{...(I)}$$

Now $m_2 g = T\cos\theta$

When θ is small, $\cos\theta = 1$

$$m_2 g = T$$

Putting in equation (I), we get

$$m_1 \ddot{x}_1 + 2Kx - m_2 g\theta = 0 \qquad \text{...(II)}$$

$$m_2 \ddot{x}_2 = -T\sin\theta$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) = -T\theta \qquad [x_2 = x_1 + l\theta]$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) + T\,\theta = 0$$

$$m_2 (\ddot{x}_1 + l\,\ddot{\theta}) + m_2 g\,\theta = 0$$

$$l\,\ddot{\theta} + g\,\theta = -\ddot{x}_1 \qquad \text{...(III)}$$

$$\omega^2 = \frac{\dfrac{m_1 g + m_2 g + 2Kl}{m_1 l} \pm \sqrt{\left(\dfrac{m_1 g + m_2 g + 2Kl}{m_1 l}\right)^2 - \dfrac{4 \times 2\,Kg}{m_1 l}}}{2}$$

$$\omega^2 = \frac{m_1 g + m_2 g + 2Kl \pm \sqrt{(m_1 g + m_2 g + 2Kl)^2 - 8\,Kg\,m_1 l}}{2m_1 l}$$

$$\omega = \left[\frac{m_1 g + m_2 g + 2Kl \pm \sqrt{(m_1 g + m_2 g + 2Kl)^2 - 8\,Kg\,m_1 l}}{2m_1 l}\right]^{1/2}$$

Let $x_1 = A \sin \omega t$; $\ddot{x}_1 = -A\,\omega^2 \sin \omega t$

$\theta = \phi \sin \omega t$; $\ddot{\theta} = -\phi\,\omega^2 \sin \omega t$

Equation (II) becomes

$$-m_1 \omega^2 A + 2KA - m_2 g \phi = 0$$

$$A\,(2K - m_1 \omega^2) - (m_2 g)\,\phi = 0$$

$$\frac{A}{\phi} = \frac{m_2 g}{2K - m_1 \omega^2} \qquad \ldots(i)$$

Equation (III) becomes

$$-\,l\,\phi\,\omega^2 \sin \omega t + g\,\phi \sin \omega t = A\,\omega^2 \sin \omega t$$

$$\phi\,(g - l\omega^2) = A\omega^2$$

$$\frac{A}{\phi} = \frac{g - l\omega^2}{\omega^2} \qquad \ldots(ii)$$

$$\frac{A}{\phi} = \frac{m_2 g}{2K - m_1 \omega^2} = \frac{g - l\omega^2}{\omega^2}$$

The frequency equation can be written as

$$m_2 g\omega^2 = 2Kg - 2Kl\omega^2 - m_1 g\omega^2 + m_1 l\omega^4$$

$$m_1 l\omega^4 - (m_1 + m_2)\, g\omega^2 - 2Kl\omega^2 + 2Kg = 0$$

$$\omega^4 - \frac{[m_1 g + m_2 g + 2Kl]\,\omega^2}{m_1 l} + \frac{2Kg}{m_1 l} = 0$$

---

7. **Calculate the natural frequency of a shaft of diameter 10 cm and length 300 cm carrying two discs of diameters 125 cm and 200 cm respectively at its ends and weighing 480 N and 900 N respectively. Modulus of the rigidity of the shaft may be taken as 2 x 1011 N/m2.**

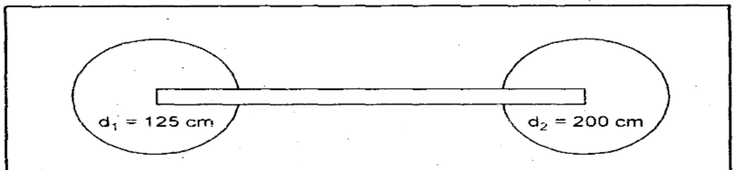


Fig. 5.23

l = 300 cm = 3 m
d = 10 cm = 0.1 m
w_1 = 480 N
w_2 = 900 N
$C = 2 \times 10^{11}$ N/m^2

$$I_1 = \frac{w_1}{g}\frac{r_1^2}{2} = \frac{480}{9.81} \times \left(\frac{62.5}{100}\right)^2 \times \frac{1}{2} = 9.56 \text{ kg.m}^2$$

$$I_2 = \frac{w_2}{g}\frac{r_2^2}{2} = \frac{900}{9.81} \times \left(\frac{100}{100}\right)^2 \times \frac{1}{2} = 45.87 \text{ kg.m}^2$$

$$K_T = \frac{CJ}{l}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(.10)^4 = 9.81 \times 10^{-6} \text{ m}^4$$

l = 3 m

$$K_T = \frac{2 \times 10^{11} \times 9.81 \times 10^{-6}}{3}$$

$K_T = 6.54 \times 10^5$ Nm/rad

$$w_n = \sqrt{\frac{K_T(I_1+I_2)}{I_1 I_2}} \text{ rad/s}$$

$$= \sqrt{\frac{6.54 \times 10^5 (9.56 + 45.87)}{9.56 \times 45.87}}$$

$$w_n = 287.52 \text{ rad/s}$$

$$f_n = \frac{w_n}{2\pi} = \frac{287.52}{2\pi} = 45.78 \text{ Hz}$$

---

8. What is co-ordinate coupling? Determine the natural frequencies of such system with dynamic coupling?

Ans. Co-ordinate coupling. When we apply brakes on a automobile two motions of car body occur simultaneously.

(1) Translatory (x)

(2) angular.

This type of unbalance occurs on thesystem because centre of gravity (C) of carand centre of rotation do not coincide.

m—Mass of car

I-*MOI

x — Translatory motion

0 — Angular Motion

Equation of motion can be written as :

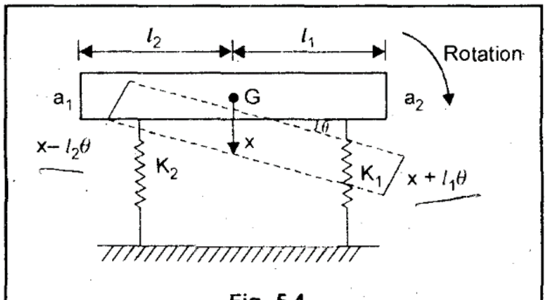


Fig. 5.4

$$m\ddot{x} = -K_2(x - l_2\theta) - K_1(x + l_1\theta)$$

$$m\ddot{x} + (K_1 + K_2)x - (K_2 l_2 - K_1 l_1)\theta = 0$$

$$I\ddot{\theta} = K_2(x - l_2\theta)l_2 - K_1(x + l_1\theta)l_1$$

$$I\ddot{\theta} = (K_2 l_2 - K_1 l_1)x + (K_2 l_2^2 + K_1 l_1^2)\theta = 0 \qquad ...(II)$$

= n (I) and (II) are coupled equations as both equations contain x and 0 terms
If

$$K_1 l_1 = K_2 l_2 \text{ then ;}$$

$$m\ddot{x} + (K_1 + K_2)x = 0 \quad \text{...(III)} \quad \text{[complete translatory equation]}$$

$$I\ddot{\theta} + (K_2 l_2^2 + K_1 l_1^2)\,\theta = 0 \quad \text{...(IV)} \quad \text{[oscillatory equation]}$$

Equation III is of translator nature.
Equation IV is of oscillator nature.

These are uncoupled differential equations and when $K_1 l_1 = K_2 l_2$ then it is called dynamic coupling.
The natural frequencies of the system are:

$$\omega_1 = \sqrt{\frac{K_1 + K_2}{m}} \text{ rad/s}, \quad \omega_2 = \sqrt{\frac{K_1 l_1^2 + K_2 l_2^2}{I}} \text{ rad/s}$$

---

9. **What are vibration absorbers ? Prove that spring force of the absorber system is equal and opposite to the excitition force for main system to be stationary?**

Ans. Vibration Absorber. When a structure which is excited by an external harmonic force has undesirable vibrations, it becomes necessary to eliminate them by coupling some vibrating system to it. The vibrating system is known as vibration absorber or dynamic vibration absorber. Vibration absorbers are used to control the structural resonance (consider the main figure)

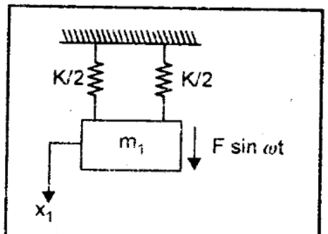


Fig. 5.5 Main System

The natural frequency of this system is $\sqrt{\frac{K}{m_1}}$. When forcing frequency (0 becomes equal to natural frequency of main system then resonance takes place. In order to reduce the amplitude of mass 'nz1' it is coupled with spring mass system (m2 — K2) called Vibration absorber. The spring mass system (ni2 — K2) will acts as vibration absorber andies the amplitude of ni1 to zero if its natural frequency is equal to the excitation frequency

$$\omega = \sqrt{\frac{K_2}{m_2}}$$

Then, when $\frac{\mathbf{K_1}}{\mathbf{m_1}} = \frac{\mathbf{K_2}}{\mathbf{m_2}}$ the absorber is called **tuned absorber.**

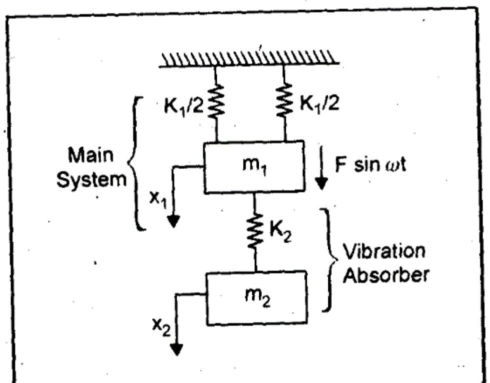


Fig 5.6

Equations of Motion

$$m_1 \ddot{x}_1 = -K_1 x_1 - K_2 (x_1 - x_2) + F \sin \omega t$$

$$m_1 \ddot{x}_1 + (K_1 + K_2)x_1 - K_2 x_2 = F \sin \omega t \quad \text{...(I)}$$

$$m_2 \ddot{x}_2 = - K_2 (x_2 - x_1)$$

$$m_2 \ddot{x}_2 + K_2 (x_2 - x_1) = 0 \quad \text{...(II)}$$

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin \omega t$$

$$m_1 \times - \omega^2 A_1 \sin \omega t + (K_1 + K_2) A_1 \sin \omega t - K_2 A_2 \sin \omega t = F \sin \omega t$$

$$A_1 (K_1 + K_2 - m_1 \omega^2) - K_2 A_2 = F \quad \text{...(III)}$$

$$m_2 \times - A_2 \omega^2 \sin \omega t - + K_2 (A_2 - A_1) \sin \omega t = 0$$

$$- K_2 A_1 + (K_2 - m_2 \omega^2) A_2 = 0 \quad \text{...(IV)}$$

Solving (III) and (IV) for A_1 and A_2. From (IV)

$$A_2 = \frac{K_2 A_1}{K_2 - m_2 \omega^2}$$

Putting in (III)

$$A_1(K_1 + K_2 - m_1\omega^2) - \frac{-K_2^2 A_1}{K_2 - m_2\omega^2} = F$$

$$A_1 [K_1 + K_2 - m_1\omega^2] [K_2 - m_2\omega^2] - K_2^2 A_1 = F (K_2 - m_2 \omega^2)$$

$$A_1 [K_1 K_2 - K_1 m_2\omega^2 + K_2^2 - m_2 \omega^2 K_2 - m_1 K_2 \omega^2 + m_1 m_2 \omega^4 - K_2^2] = F (K_2 - m_2 \omega^2)$$

$$\boxed{A_1 = \frac{F(K_2 - m_2\omega^2)}{\beta}} \quad \text{...(V)}$$

Where

$$\beta = [m_1 m_2 \omega^4 - [m_1 K_2 + m_2 (K_1 + K_2)\omega] + k_1 k_2]$$

$$\boxed{A_2 = \frac{FK_2}{\beta}} \quad \text{...(VI)}$$

In order that amplitude of mass ni1 is zero

Put A1 =0 (so that mass rn1 must not vibrate)

$$K_2 = m_2 \omega^2$$

$$\omega = \sqrt{\frac{K_2}{m_2}} = \omega_2$$

The vibration absorber in which mass and spring constant are selected such that the above condition is satisfied becomes **dynamic vibration absorber.**

Let us assume

$$A_{st} = \frac{F}{K_1} = \text{Static deflection or zero frequency deflection}$$

$$\omega_1 = \sqrt{\frac{K_1}{m_1}} = \text{Natural frequency of main system}$$

$$\omega_2 = \sqrt{\frac{K_2}{m_2}} = \text{Natural frequency of vibration absorber}$$

$$\mu_1 = \frac{m_2}{m_1} = \text{Mass ratio}$$

Multiply N^r and D^r *by* $K_1 K_2$

$$A_1 = \frac{F(K_2 - m_2\omega^2)}{K_1 K_2} \Big/ \frac{[m_1 m_2 \omega^4 - [m_1 K_2 + m_2(K_1 + K_2)]\omega^2] + K_1 K_2]}{K_1 K_2}$$

$$A_1 = \frac{F/K_1\left(1 - \frac{\omega^2}{\omega_2^2}\right)}{\frac{\omega^4}{\omega_1^2 \omega_2^2} - \left[\frac{m_1}{K_1} + m_2\left(\frac{1}{K_2} + \frac{1}{K_1}\right)\right]\omega^2 + 1}$$

$$\frac{A_1}{A_{st}} = \frac{1 - \frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2 \omega_2^2} - \left(\frac{m_1}{K_1} + \frac{m_2}{K_2} + \frac{m_2}{K_1}\right)\omega^2 + 1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{w_1^2\omega_2^2} - (\omega_1^{-2}+\omega_2^{-2}+\mu\omega_1^{-2})\omega^2+1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VII)}$$

Simlilarly

$$\frac{A_2}{A_{st}} = \frac{K_1K_2}{\beta} \qquad \left[A_2 = \frac{F}{K_1}\frac{K_1K_2}{\beta} \text{ where } \frac{F}{K_1} = X_{st}\right]$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VIII)}$$

when A1 0, from VII

$$\boxed{\omega = \omega_2}$$

At $A_1 = 0, A_2 = ?$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+1\right]+1}$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - (1+\mu)\frac{\omega^2}{\omega_1^2}} = \frac{1}{-\mu\frac{\omega^2}{\omega_1^2}}$$

$$\frac{A_2}{A_{st}} = \frac{-\omega_1^2}{\mu\omega^2}$$

$$A_2 = A_{st} \times -\frac{\omega_1^2}{\mu\omega^2} = A_{st} \times \frac{-\omega_1^2}{\mu\omega_2^2}$$

$$A_2 = \frac{-F}{K_1} \frac{K_1 m_2}{m_1 \mu K_2} = \frac{-F}{K_2} \frac{\mu}{\mu} \qquad [\because \omega = \omega_2]$$

$$\left[\mu = \frac{m_2}{m_1}\right]$$

$$\boxed{F = -A_2 K_2}$$

Hence when the amplitude A1 = 0 i.e. main system becomes stationary the spring force of the absorber is equal and opposite to exciting force. The energy of the main system is absorbed by vibration absorber which is also called auxiliary system.

Amplitude of the auxiliary system is inversely proportional to spring constant ‘K2’. This equation is used for design of absorber.
